The Social Value of Information with an Endogenous Public Signal

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Abstract

I analyse the equilibrium and welfare properties of an economy characterised by uncertainty and payoff externalities, in a general model which nests several applications. Agents receive a private signal and an endogenous public signal, which is a noisy aggregate of individual actions. I analyse how endogenous public information, which causes an information externality, combines with payoff externalities in order to disentangle their joint effect on the agents' use of signals. I find that agents underweight private information in a larger payoff parameter region compared to when public information is exogenous. Furthermore, with endogenous public information I find that the sign of the social value of private information may be overturned and that it is empirically more plausible that increasing the precision of the noise in the public signal decreases welfare in some applications, such as in the beauty contest, thus contributing to the transparency debate.

Keywords: Endogenous public information, externalities, welfare analysis.

JEL codes: D62, D82, G14.

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1 Introduction

Expectations play a key role in economies characterised by uncertainty and payoff externalities. Public predictions are influenced by expectations. In today's information age, data collection, processing and aggregation are becoming prevalent, and non-price mechanisms which combine dispersed information are widespread[1]. Disclosed public statistics, which are noisy, increasingly reflect the aggregate action rather than fundamentals, as reflected in macroeconomic statistics, internet search engine results and consensus forecasts. In other words, public information is endogenous. In addition, there is ample evidence that firms increasingly use strategies which are contingent on non-price aggregate statistics that accumulate dispersed information. For example, algorithmic trading strategies are contingent on financial and macroeconomic forecasts; marketing and production strategies progressively use predictions from social media and search engines; and airline pricing strategies use forward indicators of future sales[2].

In this paper, I explore the roles of private and endogenous public information in economies with a large number of agents. Morris and Shin (2002) and Angeletos and Pavan (2007) have studied these economies with exogenous public information. How do their results change when I consider that public information can be understood as the noisy aggregate action of the population? In particular, how do agents optimally use private and public information sources? And, what are the welfare properties of the equilibrium allocation? Finally, what are the effects of varying the precision of the noise in public and private information on welfare? I analyse the inefficiencies that may present in the equilibrium allocation and how endogenous public information combines with payoff externalities in order to disentangle their joint effect on the social value of information. The framework is general and can incorporate a number of relevant applications such as: the beauty contest, competition in a homogeneous product market, and Cournot and Bertrand competition with product differentiation.

I address these questions by using a framework based on the payoff structure of Angeletos and Pavan (2007). The model is static and of the linear-quadratic Gaussian family. The economy is populated by a continuum of agents. Each agent's utility function depends on fundamentals, such as demand or cost shocks, and exhibits payoff externalities: strategic complementarity or substitutability. Agents have access to private information as a result of their observation of local market conditions, private communications or local interactions. Each agent can also condition on an endogenous public signal, which is a noisy aggregate

[1] See, for example, Einav and Levin (2014).

[2] For algorithmic trading, see for example Chaboud et al. (2014). For marketing strategies based on social media predictions, see for example Naylor et al. (2012). For pricing in the airline industry see for example McAfee and Te Velde (2006).

of individual actions. I obtain the Bayesian Nash Equilibrium of this economy. The welfare benchmark is the ex ante utility of a team of agents, subject to the constraint that private information cannot be transferred from one agent to another, or to a centre.

The unique linear Bayesian Nash equilibrium strategy exhibits the following property. When public information aggregates the economy's dispersed information, the precision of the endogenous public statistic depends positively on the agents' response to private information, which influences how agents form expectations about fundamentals and aggregate statistics. Agents in equilibrium do not take into account that their response to private information affects the informativeness of the public signal, which increases non-fundamental volatility and augments welfare losses. Due to the information externality, I find, first, that agents place an inefficiently low weight on private information in a larger payoff parameter region compared to exogenous public information, and second that the efficient weight given to private information is larger with endogenous than with exogenous public information. Besides information externalities, there may also be payoff externalities. For each application, I characterise the regions where agents over-, under- or equally weight private information in relation to the efficient strategy.

The results regarding the social value of public information with an endogenous public signal show that a increasing the precision of the noise in the public signal decreases the weight agents give to private information, which decreases the overall informativeness of the public signal, which in turn increases non-fundamental volatility. If the full information equilibrium strategy is efficient then reducing the variance of the noise in public information has a positive social value if the reduction in dispersion is larger than the increase in non-fundamental volatility. Otherwise, the precise sign of the welfare effect depends on a combination of payoff relevant parameters and on the ratio of public to private information precisions. For example, increasing the precision of the noise in the public signal is always beneficial in firms competing in a homogeneous product market. However, increasing the precision of the noise in the public signal may be detrimental in the beauty contest if the degree of strategic complementarity is large and the ratio of overall public to private precisions is small.

Comparing the social value of public information with endogenous and exogenous public information, I obtain that the rate at which welfare changes with respect to the precision of the noise in the public statistic is slower with endogenous than with exogenous public information if the equilibrium full information response to the fundamentals is less than or equal to one in absolute value (e.g. beauty contest, monopolistic competition à la Cournot with total producer surplus as welfare benchmark). The implication for public policy is that in order to achieve a certain welfare increase, the precision of the noise in public information needs to change more with endogenous than exogenous public information. Second, I find that the ratio of public to private precisions is larger with exogenous than with endogenous public information, which has implications for the transparency debate between Morris and Shin (2002) and Svensson (2006) in the beauty contest. With exogenous public information, Morris and Shin (2002) highlight the potential detrimental effects of public information, while Svensson (2006) argues that the conditions for which public information is detrimental are empirically implausible since the precision of the noise in the private signal should be at least 8 times more precise than the noise in the public signal. Svensson (2006) claims that as central banks and information agencies devote far greater resources to analysing and processing public information than private agents do, it seems empirically implausible that the conditions for which public information are detrimental are observed in reality. With endogenous public information, I find that public information is detrimental whenever the precision of the noise in the private signal is at least 3.23 times more precise than the noise in the public signal. Therefore, endogenous public information makes it empirically more plausible that public information is detrimental, thus strengthening the conclusion of Morris and Shin (2002) in the transparency debate.

Turning to the results on the social value of private information, I find that the total welfare result of increasing the precision of the noise in private signal is the sum of two effects. First, the *'partial effect'*. Keeping the precision of the noise in public signal fixed and if the full information equilibrium strategy is efficient then increasing the precision of the noise in the private signal is beneficial for welfare if the increase in dispersion is smaller than the reduction in non-fundamental volatility. Second the *'endogenous public precision effect'*, whereby increasing the precision of the noise in the private signal increases the precision of the endogenous public statistic, which is described by the social value of public information results. I find that when both effects have the same sign then the sign of the total welfare effect is unambiguous. This occurs when firms compete in a homogeneous product market. Importantly, I find that compared to when public information is exogenous, increasing the precision of the noise in the private signal may overturn the sign of the total welfare effect if the *'partial effect'* has a different sign from the *'endogenous public precision effect'*. This may occur in monopolistic competition à la Cournot and à la Bertrand with product differentiation.

I also find the necessary and sufficient conditions for equilibrium welfare to increase with a reduction in the noise of the private signal, which depend on a combination of payoff parameters and on the ratio of public to private information precisions. Moreover, I find that the payoff parameter combination that determines the social value of private information is different with exogenous than endogenous public information due to the *'endogenous public precision effect'*. As a result, endogenous public information also changes the cutoff degree of strategic complementarity that determines whether the total welfare effect of increasing the precision of the noise in the private signal is positive or negative.

An essential mechanism of this paper is the information externality, which was first studied in the literature of social learning, such as in Banerjee (1992), Bikhchandani et al. (1992), and Vives (1993). Their main contribution is to provide a rational explanation of herding focusing on the insufficient response to private information in agents' decisions. In contrast to this paper, herding models are typically dynamic and do not consider payoff externalities. Information externalities have also been studied in the literature of rational expectations (e.g. Grossman and Stiglitz (1980), Diamond and Verrechia (1981)), where agents learn from prices. In contrast to this literature, I consider that the endogenous public statistic does not directly affect an agent's payoff and therefore it does not have an allocation role. It has only an information role. This is motivated by the empirical observation, as suggested above, that non-price procedures to aggregate information are common. In addition, this paper presents a theoretically parsimonious way to introduce information externalities which are independent of the market structure, as in herding models.

This paper contributes to the literature which studies the use of private and public information in linear-quadratic-normal type games. The seminal paper of Morris and Shin (2002) studied these issues in the beauty contest; Hellwig (2005) focused on monopolistic competition à la Bertrand with product differentiation; and Angeletos and Pavan (2007) proposed a general model which could encompass many of the previous applications. All the previous papers considered that public information was exogenous. In relation to this literature, I introduce an endogenous public signal and find that agents in equilibrium underweight private information in a larger payoff parameter region compared to when public information is exogenous due to the information externality. For example, in the beauty contest with endogenous public information, agents may underweight private information when actions are strategic substitutes, which could not occur if public information was exogenous (e.g. Morris and Shin (2002), Angeletos and Pavan (2007)); or in competition à la Bertrand with product differentiation, agents can underweight private information when actions are strategic complements, which reverses the results obtained with exogenous public information (e.g. Hellwig (2005), Angeletos and Pavan (2007)).

A related stream of literature considers the welfare effects of disclosing more precise public or private information. Considering that public information is exogenous, Ui and Yoshizawa (2015) provide a categorisation of the social value of information in quadratic games which contains the previous results (e.g. Morris and Shin (2002), Angeletos and Pavan (2007)). In relation to this literature, I find that the introduction of an endogenous public signal can overturn some previous results regarding the social value of private information due to the way in which payoff externalities combine with the information externality. These results are related to the paper of Colombo, Femminis and Pavan (2014), in which they introduce endogenous private information through information acquisition, and find that the magnitude and sign of the social value of public information may be overturned. In contrast, I focus on how endogenous public information affects the social value of private information. Furthermore, endogenous public information modifies the rate at which welfare changes with respect to the precision of the noise in public information, thus having implications for the social value of public information and the transparency debate.

The following literature also recognises that public information is endogenous and discusses the welfare effects of disclosing a more precise public signal. Morris and Shin (2005) were among the first to show that, in economies that can be described by the beauty contest, public signals are less informative when central bankers disclose their forecasts than when they do not disclose anything. I obtain the same result in a static model, and in addition extend their findings by incorporating a welfare analysis in a general class of quadratic economies that exhibit payoff externalities. Amador and Weill (2010) consider a micro-founded macroeconomic model in which prices constitute an endogenous source of information. In a model with no payoff externalities, they find that more precise public information can be detrimental. In contrast to them, I find that if there are no payoff externalities, more precise public and private information are both beneficial. In Amador and Weill (2010) there is an additional source of strategic complementarity, generated by the way agents learn from prices, which is responsible for the negative welfare result.

The paper is organised as follows. Section 2 describes the model. Section 3 studies the equilibrium and efficient allocations. Section 4 analyses the comparative statics of equilibrium welfare. Section 5 illustrates the results in several applications. Section 6 concludes. Proofs are derived in the Appendix.

2 The model

I present a static linear-quadratic-Gaussian model, based on the preference structure of Angeletos and Pavan (2007). The information structure however differs. Public information arises from imperfectly aggregating the individual decisions of each agent in the economy, and is therefore endogenous.

2.1 Preferences

The economy is composed of a continuum[3] of agents distributed uniformly over the unit interval and indexed by i. Simultaneously, each agent chooses an action, q_i, to maximise the utility function

$$U(q_i, Q, \sigma_q, \theta), \tag{1}$$

where θ represents the fundamentals which exogenously affect an agent's utility function; Q is the average of agents' actions given by $Q = \int q_i di$; and σ_q^2 is the variance of the agents' actions across the population defined as $\sigma_q^2 = \int (q_i - Q)^2 di$. Assume that U is a quadratic polynomial[4] in the variables q_i, Q, σ, θ. The dispersion in actions, σ, has only second-order effects, *i.e.* $U_{q\sigma} = U_{Q\sigma} = U_{\theta\sigma} = 0$ and $U_\sigma(q_i, Q, 0, \theta) = 0$ for all (q, Q, θ) and that U is symmetric across agents. Furthermore, I assume that the utility function is concave with respect to q_i. Without loss of generality, let

$$\alpha = -\frac{U_{qQ}}{U_{qq}} \tag{2}$$

be the degree of strategic complementarity. Actions are strategic complements whenever $\alpha > 0$, strategic substitutes whenever $\alpha < 0$ and strategically independent whenever $\alpha = 0$. To ensure uniqueness, I assume that $\alpha < 1$. Note that the payoff function described above admits payoff externalities with respect to the mean action whenever $U_Q \neq 0$ and with respect to the dispersion of actions whenever $U_\sigma \neq 0$.

2.2 Information, Timing & Equilibrium Concept

The model is static and the information structure is symmetric. First the fundamentals, θ, are drawn according to the prior distribution $\theta \sim N(\bar{\theta}, \sigma_\theta^2)$, with mean $\bar{\theta}$ and variance σ_θ^2. Each agent surveys local market conditions and forms a private belief about fundamentals given by $s_i = \theta + \epsilon_i$, where $\epsilon_i \sim N(0, \sigma_\epsilon^2)$. Simultaneously, agents have access to a noisy[5] endogenous public signal given by

$$w = \int q_i di + u, \tag{3}$$

[3] In market contexts, this assumption is as in monopolistic competition models. Ui and Yoshizawa (2015) consider a similar model with a finite number of agents.

[4] U can also be viewed as a second order approximation of a concave function.

[5] The noise in the public signal means that the average action does not fully reveal fundamentals.

where the noise in the endogenous public signal is normally distributed with $u \sim N(0, \sigma_u^2)$. Fundamentals and error terms of private and public signals are mutually independent and identically distributed across agents. There is noise in the public statistic because, for example, the aggregation process is noisy, or there is measurement error, or the information agency aggregates strategies of rational agents and noise agents that based their strategies on non-informational reasons. Then, each agent sets a strategy, $q(s_i, w)$, which is a function from the signal space to the action space of each agent. Finally payoffs are collected.

For ease of interpretation, I shall often work with the precision of a random variable, which is the inverse of its variance. For any random variable *e.g.* x, with a non-zero variance, its precision is given by $\tau_x = 1/\sigma_x^2$. Throughout the text, I call τ_ϵ the precision of the noise in the private signal, τ_u the precision of the noise in the public signal and τ_θ the precision of fundamentals.

The equilibrium concept used is Bayesian Nash Equilibrium. First, a strategy is conjectured for each agent. Given the linear-quadratic-Gaussian structure of the model, I focus on linear strategies. Second, beliefs about fundamentals are updated using conditional expectations using these conjectured strategies. Third, the conjectured strategy must be self-fulfilling, thus I find the fixed point of the best responses. These determine the coefficients of the equilibrium strategy. Finally, the precision of the public statistic is determined.

2.3 The Endogenous Public Signal

The paper focuses on the analysing the effects of an endogenous public signal which emerges from within the system. This is in contrast with the seminal papers of Morris and Shin (2002) and Angeletos and Pavan (2007) that have considered that the public signal is exogenous, defined as $y = \theta + v$, where $v \sim N(0, \sigma_v^2)$. The case of exogenous public information will serve as a benchmark.

I present three arguments which distinguish different environments that are characterised by either endogenous or exogenous public information. The first focuses on the connection between the market structure and the type of information. Avdjiev, McGuire and Tarashev (2012) argue that public information is exogenous in segmented markets, where price discovery is slow, while information is endogenous in integrated markets, where price discovery is fast. The second is related to whether the institution which discloses public information observes the market activity (endogenous) or whether directly observes the fundamental (exogenous). Examples of this approach can be found in Morris and Shin (2005), Baeriswyl (2011) and Bond and Goldstein (2015). The third draws upon a forecasting interpretation. Suppose that the information agency which discloses public information to all agents in the economy can be of two types: The first uses a 'fundamental forecasting model', whereby it releases predictors based on an econometric model whose inputs are exogenous predictors of the fundamentals, and therefore, the public signal is exogenous. The second uses a 'poll forecasting model', whereby it predicts the agents' aggregate action by compiling interviews, polls and surveys, and thus discloses an endogenous public signal [6].

Next, I provide two additional theoretical motivations for the paper's formulation of the endogenous public signal. First, notice that due to the static nature of the model, the public statistic aggregates agents' dispersed actions and simultaneously influences these actions through its information role. This is reminiscent of models which use the Rational Expectations Equilibrium (REE) concept since an agent forecasts the future according to the correct distribution of future events. Consequently, an agent's expectations are equal to the true statistical expected value conditional on the agent's information set, which includes the endogenous public signal. The classic framework of REE extensively identifies the price with the endogenous public signal, which has both information and allocation roles (e.g. starting from Grossman and Stiglitz (1980), Diamond and Verrechia (1981), and further developed by many others). The more recent papers of Vives (2011, 2013) apply this concept to market games where firms compete in demand or supply schedules (e.g. Klemperer and Meyer (1989)). In contrast to the models previously mentioned, in this paper I consider that the public statistic only has an information role, and not an allocation role, since it does not affect an agent's utility function. Then, an agent's strategy, $q(s_i, \omega)$, can be rationalised as a schedule: each agent submits a schedule which is contingent on the public statistic from the signal received. An information agency aggregates the strategies of all agents in the economy and forms the public statistic[7].

Furthermore, several papers have shown that the main properties of related fully specified dynamic models are preserved in the steady state (or static counterparts). Examples of such models can be found in Morris and Shin (2005), with a special focus on the beauty contest application; in Angeletos and Werning (2006) who study global games and equilibrium selection; the models summarised in Vives (2008); and in Angeletos and Pavan (2009) who focus on contingent taxation. In relation to these models, I focus on the social welfare implications of disclosing more precise public or private signals in a general class of quadratic models where public information is best described as endogenous.

[6] This distinction is often used in the forecasting literature. For example, see Hendry and Ericsson (2003).

[7] Public statistics may be disclosed by private or public institutions, such as the central bank, a statistical research agency or information company

3 Equilibrium and Efficiency

Considering that public information is endogenous, first I solve an agent's maximisation problem and find the equilibrium strategy. Second, I find the efficient strategy, or equivalently, the optimal weights that a welfare planner who maximises ex ante social welfare would give to public and private information. Third, I compare the efficient and equilibrium weights to the two sources of information with exogenous and endogenous public information.

3.1 Equilibrium

Each agent maximises expected utility conditional on the agent's own information set

$$max_{q_i} E\left[U(q_i, Q, \sigma_q, \theta)|s_i, w\right]. \tag{4}$$

Since the information structure is symmetric, I look for symmetric strategies. Therefore, an agent's strategy can be written as $q(s_i, w) = b' + as_i + c'w$, which is contingent on both the private signal and the endogenous noisy public statistic, w. Using the definition of the public statistic w, I note that $w = \int q_i + u = b' + a\theta + c'w + u$, whose informational content is given by the random variable, $z = a\theta + u$, so that $E\left[\theta|s_i, w\right] = E\left[\theta|s_i, z\right]$. Therefore, an agent's strategy can be written as

$$q(s_i, z) = b + as_i + cE\left[\theta|z\right]. \tag{5}$$

Importantly, the precision of the endogenous public signal[8] is

$$\tau = \left(var\left[\theta|z\right]\right)^{-1} = \tau_\theta + a^2\tau_u. \tag{6}$$

The precision of the public signal depends quadratically on the response an agent gives to private information. The more agents respond to private information, the more informative the public signal becomes. The results of the paper cannot be understood without taking into account the feedback effect between an individual strategy and the precision of the endogenous public signal. This effect is not present when public information is exogenous. Proposition 1 spells out the equilibrium strategy.

[8] τ is often referred throughout the text as the overall precision of the endogenous public signal. This is done to distinguish it from τ_u which is the precision of the noise in the public signal.

Proposition 1 (Equilibrium). *For finite and non-zero precisions of information and* $\alpha < 1$*, there exists a unique linear Bayesian Nash Equilibrium, which is given by* $q(s_i, z) = b + as_i + cE[\theta|z]$*, where* $a = k_1\gamma$*,* $b = k_0$*,* $c = k_1(1-\gamma)$ *and* k_0, k_1 *define the full information equilibrium strategy given by* $k(\theta) = k_0 + k_1\theta$*, where* $k_0 = \frac{-U_q(0,0,0,0)}{U_{qq}+U_{qQ}}$ *and* $k_1 = \frac{-U_{q\theta}}{U_{qq}+U_{qQ}}$*. The equilibrium weight given to private information,* γ^m*, is the unique real solution to a cubic equation which can be written implicitly as*

$$\gamma = \frac{(1-\alpha)\tau_\epsilon}{(1-\alpha)\tau_\epsilon + \tau_\theta + k_1^2\tau_u\gamma^2}, \tag{7}$$

where γ^m *satisfies* $0 < \gamma^m < 1$.

Henceforth, a is the equilibrium response to private information, while γ is the equilibrium weight given to private information. Similarly for c and $1-\gamma$ with respect to public information.

If actions were strategically independent, the weights given to public and private information would correspond to Bayesian weights according to the relative precisions of the signals. When actions are strategic complements, agents place a higher weight on the public signal since it helps to predict better the aggregate action. The converse happens when actions are strategic substitutes. Endogenous public information modifies the precision of the public statistic in relation to the case of exogenous public information since a higher weight given to private information makes the public statistic more informative. The precision of the noise in the public signal affects the size of the Bayesian weight, which modifies the equilibrium weights an agent gives to public and private information.

When the precision of the noise in the private signal tends to zero, agents do not give any weight to the private signal. As the precision of this signal increases, agents increase the weight given to the private signal until they place all the weight on the private signal, corresponding to an infinite precision of it. The weight given to the private signal decreases with the precision of fundamentals and with the precision of noise in the public statistic. At the limit, when the noise in the public signal is infinitely precise, agents give all the weight to the endogenous public statistic. These comparative statics are summarised in the corollary below.

Corollary 1. *The equilibrium weight given to the private signal increases with the precision of the noise in the private signal, while it decreases with the precision of fundamentals and the precision of the noise in the public statistic. Furthermore, the equilibrium weight given to the private signal decreases with a larger degree of strategic complementarity.*

3.2 Efficiency

Using the ideas of Radner (1962), Vives (1988) and Angeletos and Pavan (2007), the welfare planner maximises ex ante utility subject to the constraint that information can be neither transferred from one agent to another, nor does the welfare planner have access to the agent's private information. A Taylor expansion of ex ante utility[9] can be written as

$$E\left[u\right] = E\left[W(k(\theta),0,\theta)\right] + E[W_Q(k(\theta),0,\theta)(Q-k(\theta))] + \frac{W_{QQ}}{2}E\left[(Q-k(\theta))^2\right] + \frac{W_{\sigma\sigma}}{2}E\left[(Q-q)^2\right], \tag{8}$$

where W is defined as $W(Q,\sigma_q,\theta) = \int U(q_i,Q,\sigma_q,\theta)di$, where $W_{QQ} = U_{qq} + 2U_{qQ} + U_{QQ}$ and $W_{\sigma\sigma} = U_{qq} + U_{\sigma\sigma}$ and assume that $W_{QQ} < 0$ and $W_{\sigma\sigma} < 0$.

The expression of ex ante utility is the sum of four terms: (1) The expected aggregate utility at the full information equilibrium strategy. Notice that this term only depends on the precision of fundamentals, τ_θ. (2) The covariance between social return to the aggregate action, W_Q, and the difference between the equilibrium aggregate action with incomplete and complete information, $Q-k(\theta)$. Notice that this term is zero when the full information action is efficient since the full information strategy, $k^*(\theta)$, is defined by $W_Q(k^*(\theta),0,\theta) = 0$. (3) The welfare loss due to non-fundamental volatility. (4) The welfare loss due to the dispersion of actions in the cross-section of the population.

If the full information equilibrium strategy is efficient, $k^*(\theta) = k(\theta)$, the ex ante utility is then a sum of the expected aggregate utility at the full information efficient strategy, $W(k^*(\theta),0,\theta)$, and welfare losses due to non-fundamental volatility and dispersion.

The relative trade-off between volatility and dispersion, $\frac{W_{QQ}}{W_{\sigma\sigma}}$, is denoted by $1-\alpha^*$, where α^* was named as the socially optimal degree of strategic complementarity (with exogenous public information) by Angeletos and Pavan (2007). Without loss of generality, throughout the text I shall generically call α^* the planner's weight on volatility relative to dispersion[10].

Comparative statics of the ex ante utility with respect to τ_ϵ, τ_u will be equivalent to the

[9] The derivation of these expressions is shown in the Proof of Proposition 2 in the Appendix.

[10] Notice that with endogenous public information, α^* is no longer the optimal degree of strategic complementarity.

opposite comparative statics to those of equilibrium welfare losses, given by

$$WL(\gamma^m) = \frac{\mid W_{\sigma\sigma} \mid (k_1)^2}{2} \left(\frac{(1-\alpha)(1-\alpha+2\phi(1-\alpha^*))\tau_\epsilon + (1-\alpha^*)(1+2\phi)\tau}{((1-\alpha)\tau_\epsilon + \tau)^2} \right), \quad (9)$$

where $\phi = \frac{k_1^* - k_1}{k_1}$ is a measure of the degree of inefficiency with full information, and is equal to zero if the full information equilibrium action is efficient.

With incomplete information, the efficient strategy, $q^*(s_i, z)$, can be found by maximising the ex ante utility. Given the structure of the model, I also focus on efficient strategies which are linear, as given by the next proposition.

Proposition 2 (Efficiency). *For finite and non-zero precisions of information and $\alpha^* < 1$, the efficient linear strategy can be written as $q^*(s_i, z) = a^* s_i + b^* + c^* E[\theta \mid z]$, where $a^* = k_1^* \gamma^*$, $b^* = k_0^*$, $c^* = k_1^*(1-\gamma^*)$, and the full information strategy is a unique linear function of fundamentals given by $k^*(\theta) = k_0^* + k_1^*\theta$, where $k_0^* = \frac{-W_Q(0,0,0)}{W_{QQ}}$* and *$k_1^* = \frac{-W_{Q\theta}}{W_{QQ}}$. The efficient weight given to private information, γ^*, is the unique real and positive root of a fifth degree equation which can be written implicitly as*

$$\gamma^* = \frac{(1-\alpha^*)\tau_\epsilon}{(1-\alpha^*)\tau_\epsilon + \tau^* - (1-\alpha^*)(1-\gamma^*)^2 \frac{\tau_u \tau_\epsilon k_1^{*2}}{\tau^*}}, \quad (10)$$

where $0 < \gamma^* < 1$ and the efficient informativeness of the public statistic is $\tau^* = \tau_\theta + (a^*)^2 \tau_u$.

Henceforth, a^* is the efficient response to private information, while γ^* is the efficient weight to private information.

Notice that as the precision of the noise in the private signal increases from zero to infinity, the efficient weight to private information increases from 0 to 1. The efficient weight to the private signal is decreasing in the precision of fundamentals and in the planner's weight on volatility relative to dispersion, α^*. However, the effects of increasing the precision of the noise in the public statistic are ambiguous, reflecting the fact that the planner in setting γ^* balances the decrease in dispersion with the increase in volatility. This is due to both the decrease in the weight given to private information and to the decrease in the precision of the public statistic. These comparative statics are summarised in the corollary below.

Corollary 2. *The efficient weight to the private signal increases with the precision of the noise in the private signal, while it decreases with the precision of fundamentals and with the planner's weight on volatility relative to dispersion, α^*. The comparative statics of the efficient weight to the private signal with respect to the precision of the noise in the public statistic are ambiguous.*

3.3 Comparison with benchmarks

Proposition 3 compares the equilibrium and efficient strategies.

Proposition 3 (Comparison of Equilibrium and Efficient Strategies). *The sign of the difference between the equilibrium and efficient weights to private information is given by:*

$$sign\,(\gamma^m - \gamma^*) = sign\left(\alpha^* - \alpha - \frac{(1-\alpha^*)(1-\alpha)(k_1^2\tau_u\tau_\epsilon)}{((1-\alpha)\tau_\epsilon + \tau)^2}\right). \tag{11}$$

There are three cases to consider:

If $\alpha = \alpha^*$ then agents always give an inefficiently low weight to private information for all values of the degree of strategic complementarity.

Otherwise, suppose that $\alpha^* = \xi\alpha$, $\xi \geq 0$ and $\xi \neq 1$.

If $\xi < 1$ then there exists a level of strategic complementarity, $\alpha^0 < 0$, such that the equilibrium weight is efficient, $\gamma(\alpha^0) = \gamma^*(\alpha^0)$, and agents over-, under-, or equally weight private information according to: $sign(\gamma - \gamma^*) = sign\,(\alpha^0 - \alpha)$.

If $\xi > 1$ then there exists a level of strategic complementarity, $\alpha^0 > 0$, such that the equilibrium weight is efficient, $\gamma(\alpha^0) = \gamma^*(\alpha^0)$, and agents over-, under-, or equally weight private information according to: $sign(\gamma - \gamma^*) = sign\,(\alpha - \alpha^0)$.

Proposition 3 allows comparison of the equilibrium and efficient strategies allows the inefficiencies that may be present in the equilibrium allocation to be identified. First, there may be a full information inefficiency if $k^*(\theta) \neq k(\theta)$. Second, in equilibrium, agents may not have the same individual incentives to align their actions as is collectively efficient (trade-off between volatility and dispersion). This inefficiency is summarised by $\alpha^* - \alpha$. Third, there is an information externality which occurs because public information is endogenous. In contrast to the welfare planner, agents in equilibrium do not take into account that a higher weight on private information increases the precision of the endogenous public statistic. The information externality vanishes if the noise in the endogenous public statistic is infinitely volatile ($\tau_u \to 0$) or if the private signal is infinitely noisy ($\tau_\epsilon \to 0$).

The magnitude of the ratio of collective to individual incentives to align actions[11], ξ, determines: (1) whether the level of strategic complementarity that internalizes both payoff and information externalities, α^0, is positive or negative; (2) whether agents over-, under- or equally respond to private information when actions are strategic complements or substitutes. The information externality is not present when public information is exogenous, and it is the key driver of the differences between the equilibrium and efficient weights to private information, as summarised in the next corollary.

Corollary 3. *i) Agents underweight private information ($\gamma^* > \gamma$) for a larger range of degrees of strategic complementarity (α) with endogenous than exogenous public information. ii) For finite and non-zero equal precisions of endogenous and exogenous public information, i.e. $\tau^* = \tau_\theta + \tau_v$, the efficient weight to private information is always larger with endogenous public information than with exogenous public information: $\gamma^* > \gamma^{*exo}$.*

The first comparison concerns the relationship between the equilibrium and efficient strategies with exogenous and endogenous public information. Due to the information externality, agents in equilibrium underweight private information in a larger payoff-parameter region (represented by a larger range of values of α) compared to when public information is exogenous. The second comparison is related to the efficient weight to private information with endogenous and exogenous public information. If the precisions of the exogenous and endogenous public statistics were the same, the efficient weight to private information would always be larger with endogenous than with exogenous public information. This is due to the fact that the welfare planner internalizes the feedback effect between the weight to private information and the precision of the endogenous public statistic, and as a result gives a higher weight to private information with endogenous than exogenous public information.

4 Social Value of Public and Private Information

In this section, I analyse the comparative statics of ex ante utility evaluated at the equilibrium strategy with respect to the information parameters. First, I discuss the social value of public information, which derives comparative statics of equilibrium welfare with respect to the precision of the noise in the public signal, τ_u. I assume that the welfare planner cannot

[11] Many applications can be described by this simplified setting (see Section 5): $\alpha^* = \xi\alpha$, $\xi \geq 0$. Equation (12) gives the general result.

change the precision of the ex ante fundamentals, τ_θ. Second, I present the results on the social value of private information, which conducts comparative statics of equilibrium welfare with respect to the precision of the noise in the private signal, τ_ϵ. Third, I compare these results to the benchmark of exogenous public information.

4.1 Social value of public information

When public information is a noisy statistic of the aggregate action, the welfare effect of changing the precision of the noise in the public signal can be written as

$$\frac{d(E[u(\gamma^m)])}{d\tau_u} = \left(\frac{\partial(E[u(\gamma^m)])}{\partial\tau}\right)_{\gamma\, cons.} \frac{d\tau}{d\tau_u}. \tag{12}$$

There are two multiplicative effects on equilibrium welfare: (1) The effect of changing the overall precision of the public signal on equilibrium welfare, keeping the weight on the private signal fixed. This is a partial effect; (2) the effect of changing the precision of the noise in the public signal on the overall precision of the endogenous public signal.

Let me discuss the first effect. Keeping the weight on the private signal fixed, the welfare effect of increasing the overall precision of the endogenous public statistic depends on the relationship between α, α^* and on the degree of inefficiency with full information, ϕ, as will be further discussed in Proposition 4.

The second effect is always positive since increasing the precision of the noise in the public signal increases the overall precision of the endogenous public statistic. This can be explained by decomposing this effect into two. A *direct effect:* For a fixed weight on the private signal, increasing the precision of the noise in the public signal (τ_u) increases the overall precision of the endogenous public signal (τ). An *indirect effect* which is only present when public information is endogenous. Increasing the precision of the noise in the public signal makes agents place less weight on the private signal, which reduces the overall precision of the endogenous public statistic. Overall, the direct effect dominates the indirect effect, and increasing the precision of the noise in the public signal increases the informativeness of the endogenous public statistic. As a result the sign of the welfare effect will be determined exclusively by the first effect, given by $\left(\frac{\partial(E[u(\gamma^m)])}{\partial\tau}\right)_{\gamma\, cons.}$. Proposition 4 summarises these considerations.

Proposition 4 (Social Value of Public Information): *For non-zero and finite precisions of $\tau_\theta, \tau_\epsilon$, equilibrium welfare increases with respect to the precision of the noise in the public signal, τ_u if and only if*

$$\eta \leq \frac{\tau}{\tau_\epsilon}\varsigma, \tag{13}$$

where $\eta = \frac{(1-\alpha)(2\alpha-\alpha^*-1-2\phi(1-\alpha^*))}{(1-\alpha^*)}$ *and* $\varsigma = (1+2\phi)$.

Proposition 4 finds necessary and sufficient conditions for equilibrium welfare to increase with the precision of the noise in the endogenous public statistic. For example, when $\phi \geq 0$, equilibrium welfare increases with a more precise public signal if $\alpha^* \geq \alpha$, since the reduction in dispersion is larger than both the increase in non-fundamental volatility and the increase in the covariance term in equilibrium welfare loss. When ϕ is sufficiently negative (i.e. agents over-respond to the fundamentals with full information), equilibrium welfare decreases with respect to τ_u. Conversely, when ϕ is sufficiently positive (i.e. agents under-respond to the fundamentals with full information), equilibrium welfare increases with respect to τ_u

The proposition shows how the welfare effects of varying the noise in the endogenous public signal depend on the ratio of public to private precisions ($\frac{\tau}{\tau_\epsilon}$), which is different with exogenous and endogenous public information; and on a combination of payoff relevant parameters, η and ς, which are invariant to the type of public information (endogenous or exogenous).

4.2 Social value of private information

The welfare planner can conduct policies that change the precision of the noise in the private signal, whose effect on equilibrium welfare can be written as

$$\frac{d(E[u(\gamma^m)])}{d\tau_\epsilon} = \left(\frac{\partial(E[u(\gamma^m)])}{\partial\tau_\epsilon}\right)_{\tau\, cons.} + \left(\frac{\partial(E[u(\gamma^m)])}{\partial\tau}\right)_{\gamma\, cons.} \frac{d\tau}{d\tau_\epsilon}. \tag{14}$$

The welfare effect of changing the precision of the noise in the private signal can be separated into two additive effects: (1) The '*partial effect*': The effect of changing the precision of the noise in the private signal on equilibrium welfare, whilst keeping the precision of the noise in the public signal fixed. (2) The *'endogenous public precision effect'*: As the precision of the noise in the private signal increases, the precision of the endogenous public signal also increases, which has an effect on equilibrium welfare.

The sign of the first effect, the '*partial effect',* is ambiguous and depends on the following. For a fixed precision of the noise in the public signal, equilibrium welfare increases with more precision in the noise in the private signal if $\phi \geq 0$ and $\alpha \geq \alpha^*$, since the increase in dispersion is smaller than the reduction in non-fundamental volatility and the reduction in the covariance term in equilibrium welfare loss. The second effect, the *'endogenous public precision effect',* is composed of two terms: a) increasing the precision of the noise in the private signal increases the overall precision of the endogenous public signal ($\frac{d\tau}{d\tau_\epsilon} > 0$); b) increasing the precision of the endogenous public signal affects equilibrium welfare, as described in Proposition 4. The next proposition summarises the welfare effects of increasing the precision of the noise in the private signal.

Proposition 5 (Social Value of Private Information): *For non-zero and finite precisions of τ_θ, τ_u, equilibrium welfare increases with the precision of the noise in the private signal if and only if*

$$\varphi\tau_\epsilon + \chi\tau \leq 0 \tag{15}$$

where $\varphi = -(1-\alpha)^2(1-\alpha+2\phi(1-\alpha^*)) + \upsilon(1-\alpha)(2\alpha-1-\alpha^*-2\phi(1-\alpha^*))$
and $\chi = (1-\alpha)(2\alpha^*-\alpha-1) - 2\phi(1-\alpha)(1-\alpha^*) - \upsilon(1-\alpha^*)(1+2\phi)$
where $\nu = \frac{d\tau}{d\tau_\epsilon} = 2k_1^2\tau_u\gamma^m\frac{d\gamma^m}{d\tau_\epsilon} > 0.$

Proposition 5 spells out the necessary and sufficient conditions for equilibrium welfare to increase with respect to the precision of the noise in the private signal. The previous proposition summarises four possible cases: (1) If welfare increases with the precision of the noise in the public signal and if the '*partial effect'* is positive, then welfare also increases with the precision of the noise in the private signal. (2) If welfare decreases with the precision of the noise in the public signal and if the '*partial effect'* is negative, then welfare also decreases with the precision of the noise in the private signal. (3) and (4) refer to cases in which the social value of public information (Proposition 4) has a different sign than the *'partial effect'* of increasing the precision of the noise in the private signal on welfare. In these two cases, it is possible that endogenous public information can overturn the previous welfare results of varying the precision of the noise in private information.

Besides changing the ratio of public to private precisions, endogenous public information also changes the payoff parameter combination that determines the social value of private information (since $\nu \neq 0$). As a result, it changes the cutoff degree of strategic complementarity that determines whether the total welfare effect of increasing the precision of the noise in private information is positive or negative.

4.3 Comparison with benchmarks

The next corollary compares the social value of information with endogenous and exogenous public information. It shows the importance of the full information equilibrium response to the fundamentals, k_1, in differentiating the effects of endogenous and exogenous public information on the social value of information. This is because k_1^2 determines the magnitude of the response to private information, which is used to form the public signal from the dispersed information in the economy.

Corollary 4. *i) With endogenous public information, if* $k_1^2 \leq 1$ *then reducing the noise in the endogenous public signal changes welfare at a slower rate compared to when public information is exogenous, i.e.* $\left| \frac{d(E[u(\gamma^m)])}{d\tau_u} \right| \leq \left| \left(\frac{\partial(E[u(\gamma^m)])}{\partial\tau_u} \right)_{\gamma\, cons.} \right|$.

ii) Compared to when public information is exogenous, increasing the precision of the noise in the private signal may overturn the sign of the total welfare effect if the 'partial effect' has a different sign from the 'endogenous public precision effect'.

iii) Whenever the noise in the endogenous public signal is the same as the noise in the exogenous public signal, the ratio of overall public to private information precisions is larger with endogenous compared to exogenous public information if and only if the response to private information is larger than one ($\frac{\tau^{exo}}{\tau_\epsilon} \geq \frac{\tau}{\tau_\epsilon} \iff 1 \geq k_1^2\gamma^2$*).*

Corollary 4 shows first that if $k_1^2 \leq 1$ then equilibrium welfare as a function of the precision of the noise in the endogenous public signal is flatter with endogenous than exogenous public information.

The second implication concerns the social value of private information. The sign of the social value of private information may be overturned with endogenous public information in relation to when public information is exogenous as can be seen from equation (14) and Proposition 5.

The third implication concerns the ratio of overall public to private precision. If $k_1^2\gamma^2 \geq 1$ ($k_1^2\gamma^2 < 1$), then the precision of the public signal is larger (smaller) with exogenous than with endogenous public information. Notice that both Proposition 4 and 5 show that welfare increases with the precision of information if and only if a certain condition is satisfied. This condition depends on parameters of the utility function (α, α^*, ϕ) and on a ratio of public to private information precision, $\frac{\tau}{\tau_\epsilon}$, which is larger with endogenous than exogenous public information if and only if $k_1^2\gamma^2 \geq 1$.

5 Applications

This section describes four applications, which illustrate how, due to differences in the payoff structure, the endogeneity of public information is reflected in the equilibrium and efficient allocations (illustrated in Figure 2), and in the social value of information (illustrated in Figure 3). These applications represent simplified settings that can be analysed with a linear-quadratic-Gaussian model. The applications often can be thought of reduced forms of more complex models, some of which are discussed at the end of each sub-section, and share the same properties as the application discussed. Table 1 shows the values of each of the three parameters which characterise the applications: the degree of inefficiency of the full information equilibrium (ϕ), the relationship between α, α^*, and the full information equilibrium response to the fundamentals, k_1^2.

Application	ϕ	α, α^*	k_1^2
Competition in a homogeneous product market[1]	0	$\alpha = \alpha^*$	$\frac{1}{(\lambda+\beta)^2}$
Competition à la Cournot with product differentiation[2]	<0	$\alpha^* < \alpha < 0$	$1 >$
Beauty Contest	0	$\alpha^* = 0,\ \alpha \in (-1, 1)$	1
Anti-Beauty Contest[3]	0>	$\alpha^* > \alpha = 0$	1

Table 1: Summary of Applications

1. Using total surplus as welfare benchmark.

2. Using producer surplus as welfare benchmark.

3. Same type of game as competition à la Bertrand with product differentiation and presents the inverse incentives as the beauty contest (hence the name).

5.1 Firms Competing in a Homogeneous Product Market

This model was first studied by Vives (1988) and further developed by Angeletos and Pavan (2007). Suppose that an economy is composed of a continuum of households, each consisting of a producer and a consumer which make production choices with quadratic production costs. Each agent is uncertain about the intercept of his own marginal cost, which is represented by fundamentals. Each household chooses quantities q_{1i}, q_{2i} of the two goods in the economy by maximising the utility given by: $u_i = \delta q_{1i} + q_{2i}$, subject to the budget constraint $lq_{1i} + q_{2i} = \pi_i$, where l is the price of good 1, good 2 is the numeraire and π_i are the profits of producer i given by: $\pi_i = (l - \theta)q_i - \frac{\lambda q_i^2}{2}$, where q_i is the quantity produced by household i. The inverse demand for good 1 is $l = \delta - \beta Q$, where Q is the total amount produced by all households in the economy. Imposing symmetry of production choices and market clearing, the original maximisation problem subject to the budget constraint is equivalent to maximising:

$$U(q, Q, \sigma_q, \theta) = (\delta - \beta Q)q_i + \frac{\beta Q^2}{2} - \theta q_i - \frac{\lambda q_i^2}{2}, \tag{16}$$

assuming that $\beta+\lambda>0$ and $2\beta+\lambda>0$. The degree of strategic complementarity is $\alpha=\frac{-\beta}{\lambda}$. In the economy, $W(Q,\sigma_q,\theta)$ is equal to total surplus since $W(Q,\sigma_q,\theta)=\int U(q,Q,\sigma_q,\theta)di=(\delta-\frac{\beta Q}{2})Q-\int(\theta q_i+\frac{\lambda}{2}q_i^2)di=\delta Q-\frac{(\beta+\lambda)Q^2}{2}-\theta Q-\frac{\lambda\sigma_q^2}{2}=TS$. With full information the economy is efficient with $k(\theta)=k^*(\theta)=\frac{\delta-\theta}{\lambda+\beta}$, and as a result $\phi=0$. Note that $\xi=1$ since $\alpha^*=\alpha=\frac{-\beta}{\lambda}$.

A trade association publishes aggregate output forecasts which constitute the source of public information. When the market is segmented, the public output forecast reflects exogenous information since the information flow is limited. When the market is integrated, the output forecast is endogenous since the trade association can aggregate individual outputs to form the public statistic. Both[12] public forecasts are noisy because aggregation is imperfect. In addition, each agent forms a private forecast based on local information. Firms' strategies are contingent on the private forecast and on the public forecast disclosed by the trade association[13]. When public information is endogenous, firms' strategies are schedules which are contingent on the endogenous public statistic from the private signal received, such as for example quantity-setting strategies which use aggregate data from exchanges as inputs. The next corollary shows the relationship between the equilibrium and efficient allocations with exogenous and endogenous public information (also refer to Figures 2a and 2b).

Corollary 5. *With exogenous public information, the equilibrium allocation is efficient. With endogenous public information, agents in equilibrium give an inefficiently low weight to private information for all degrees of strategic complementarity.*

Since there are no payoff externalities, the equilibrium allocation is efficient with exogenous public information (e.g. Angeletos and Pavan (2007)). However, when public information is endogenous and since $\alpha^*=\alpha$, agents in equilibrium give an inefficiently low weight to private information for all degrees of strategic complementarity due to the information externality (Proposition 3).

The results of Corollary 5 have been previously found in the literature with models that have the same payoff structure (i.e. $k(\theta)=k^*(\theta)$ and $\alpha=\alpha^*$) and that also consider that public information is endogenous. One example is Bru and Vives (2002) in the context of a

[12] An example of an integrated market is the market for goods traded on exchanges while a segmented market could be an over-the-counter market.

[13] This interpretation of segmented and integrated markets is based on the market microstructure and has been proposed by Avdjiev et al. (2012) in the context of the disclosure of policy signals. However, one could also provide an alternative explanation of exogenous (endogenous) public information based on the types of information flows emerging from a disconnected (connected) network.

pure prediction model[14] . Another is the business cycle framework of Angeletos and La'O (2013) which show that the model's reduced form is the same type as the one considered in this section. They show that the economy is efficient if public information is exogenous, while agents place an inefficiently low weight to the private signal, thus making employment, output and consumption choices inefficient with endogenous public information.

The result of Corollary 5 contrasts with the results of Vives (2013) where firms use price contingent strategies. In contrast to the results of this paper, he finds that firms can over-weight private information whenever actions are strategic substitutes and supply functions are upward sloping. This is due to the fact that in Vives (2013) the price has both information and allocation roles, and the result follows from the way these two roles interact, while in the model presented here, the endogenous public statistic only has an information role.

The next corollary discusses the social value of information with an exogenous and an endogenous public signal.

Corollary 6. *With both endogenous and exogenous public information, welfare increases with the precision of the noise in the public and private signals.*

Figures 3a and 3b illustrate the welfare contour lines, which summarise the social value of public and private information with exogenous and endogenous public information, respectively. Corollary 6 contrasts with the results of Amador and Weill (2010) who find that more precise public information can be detrimental in a micro-founded macroeconomic model with no payoff externalities and with prices that constitute an endogenous source of information. The difference is due to the fact that in Amador and Weill (2010) there is an additional source of strategic complementarity, generated by the way agents learn from prices, which is responsible for the negative welfare result.

The magnitude of k_1^2 matters for determining the curvature of the welfare contour lines. When $k_1^2 \leq 1$ (or equivalently $(\lambda + \beta)^2 \geq 1$), for a constant precision of the noise in private information and if $(\lambda + \beta)^2 \geq 1$ then a higher precision of the noise in the public statistic increases welfare at a slower rate with endogenous versus exogenous public information. Notice that these results may be reversed if $k_1^2 > 1$ (or equivalently $1 > (\lambda + \beta)^2$). It is important to notice that these differences are entirely due to the information externality since there are no other externalities.

[14] The utility function which describes a pure prediction model is: $U(q, Q, \sigma_q, \theta) = -(q_i - \theta)^2$, with $k(\theta) = k^*(\theta) = \theta$ and $\alpha = \alpha^* = 0$.

5.2 Monopolistic Competition à la Cournot with Product Differentiation

This formulation is based on the model of Vives (1990). Myatt and Wallace (2014) analyse a similar game with a more general information structure. Suppose a large number of firms compete à la Cournot and sell a differentiated product. The inverse demand that a firm faces is linear and random given by $l_i = \theta - (1-\delta)\kappa_i - \delta K$ where (l_i, κ_i) are price-quantity pairs for firm i, and δ can be interpreted as the degree of product differentiation with $\delta \in (0,1)$. At the limit, when $\delta \to 0$ firms are isolated monopolies, while when $\delta \to 1$ there is perfect competition. For simplicity, suppose that marginal costs are constant and equal to zero. Setting $\kappa_i = q_i$, the profit of each firm can be written as:

$$U(q, Q, \sigma_q, \theta) = \theta q_i - (1-\delta)q_i^2 - \delta q_i Q. \tag{17}$$

With full information, the profit maximising quantity is equal to $k(\theta) = \frac{\theta}{2-\delta}$ and the degree of strategic substitutability is $\alpha = \frac{-\delta}{2(1-\delta)} < 0$. Social welfare, W, corresponds to aggregate profits and it is equal to: $\Pi = W(Q, \sigma_q, \theta) = \theta Q - Q^2 - (1-\delta)\sigma_q^2$, which implies that $\alpha^* = 2\alpha < 0$. With full information, firms over-respond to the fundamentals since $k^*(\theta) = \frac{\theta}{2} < k(\theta)$. This implies that the full-information inefficiency is $\phi = \frac{-\delta}{2} = \frac{\alpha}{1-2\alpha} < 0$. Another relevant welfare is total surplus. Suppose that a representative consumer maximises $V(q_i) - \int l_i q_i di = \int (\theta q_i - \frac{Q^2}{2} - \frac{(1-\delta)(q_i^2 - Q^2)}{2} - l_i q_i) di$. Then, total surplus can be shown to be equal to $TS = \theta Q - \frac{Q^2}{2} - \frac{(1-\delta)\sigma_q^2}{2}$, where the relationship between the representative consumer's utility function and total surplus is $\alpha^* = 2\alpha$ and the full-information inefficiency is: $\phi = 1 - \delta = \frac{1}{1-2\alpha} > 0$.

An information agency publicly discloses aggregate output forecasts. Suppose that there are two types of information agencies. The first type uses an econometric model which projects output on the basis of exogenous predictors of the demand shock and discloses an exogenous public forecast. The second type uses interviews, polls and surveys of market participants to form an aggregate endogenous public statistic. Both are noisy because of measurement error. In addition, firms have access to private consultancy reports which equip them with private information. Firms set quantity strategies which are contingent on the information provided by the private consultancy report and the aggregate public forecast. When public information is endogenous these type of strategies could be understood as schedules, which are contingent on the disclosed public statistic of the second type of information agency, such as firms' strategies which use big data analytics in setting production schedules given the firm's private information. Next corollary shows the equilibrium and efficient weights to private information with exogenous and endogenous public information (also refer to Figures 2g and 2h).

Corollary 7. *Both with exogenous and endogenous public information, firms underweight private information in relation to the efficient level. The magnitude of the difference between the equilibrium and efficient weights to private information is larger with endogenous than exogenous public information.*

In Cournot competition firms underweight private information because firm's actions are strategic substitutes but less than what it is collectively optimal (i.e. $\alpha^* - \alpha < 0$). The magnitude of the difference is larger with endogenous than exogenous public information due to the information externality. The next Corollary applies Propositions 4 and 5 to the competition à la Cournot with product differentiation to both when public information is exogenous ($\nu = 0$) and endogenous ($\nu > 0$).

Corollary 8. *Case I: If $-\frac{1}{2} \leq \alpha < 0$ then total profits increase with the precision of the noise in the public and private signals.*

Case II: If $\upsilon - \sqrt{\upsilon^2 + \upsilon + 1} \leq \alpha < \frac{-1}{2}$ then

i) If $\eta \leq \varsigma \frac{\tau}{\tau_\epsilon}$ and $\frac{\varphi}{(-\chi)} \leq \frac{\tau}{\tau_\epsilon}$ then total profits increase with the precision of the noise in the public and private signals.

ii) If $\eta > \varsigma \frac{\tau}{\tau_\epsilon}$ and $\frac{\varphi}{(-\chi)} \leq \frac{\tau}{\tau_\epsilon}$ then total profits decrease with the precision of the noise in the public signal and increase with the precision of the noise in the private signal.

Case III: If $\alpha < \upsilon - \sqrt{\upsilon^2 + \upsilon + 1}$ then

i) If $\eta \leq \varsigma \frac{\tau}{\tau_\epsilon}$ and $\frac{\varphi}{(-\chi)} \leq \frac{\tau}{\tau_\epsilon}$ then total profits increase with the precision of the noise in the public and private signals.

ii) *If $\eta > \varsigma \frac{\tau}{\tau_\epsilon}$ and $\frac{\varphi}{(-\chi)} \leq \frac{\tau}{\tau_\epsilon}$ then total profits decrease with the precision of the noise in the public signal and increase with the precision of the noise in the private signal.*

iii) If $\eta > \varsigma \frac{\tau}{\tau_\epsilon}$ and $\frac{\varphi}{(-\chi)} > \frac{\tau}{\tau_\epsilon}$ then total profits decrease with the precision of the noise in the public and private signals.

Corollary 8 shows that the social value of information can be described by three cases, as illustrated in Figure 3a and 3b for Case I; to Figures 3c and 3d for Case II; and to Figures 3g and 3h for Case III with endogenous and exogenous public information, respectively. Given the parameter configuration for competition à la Cournot, it is not possible that welfare increases with the precision of the noise in public information while it decreases with the precision of the noise in private information.

The benchmark case with exogenous public information has been studied by Ui and Yoshizawa (2015), which corresponds to the results of Corollary 8 with $\nu = 0$. When public information is endogenous, the cutoff value between Case II and Case III changes in relation to when public information is exogenous. In particular, the range of α that falls in Case II is smaller with endogenous than with exogenous public information (since $\upsilon - \sqrt{\upsilon^2 + \upsilon + 1} > -1$ for $\upsilon > 0$). In addition, endogenous public information changes the payoff parameter combination that determines the social value of private information ($-(1+\alpha)$ with exogenous vs. $\frac{-((1-\alpha)^2(1+\alpha)+\upsilon(1-\alpha)(1+2\alpha))}{\upsilon+(1-\alpha)^2}$ with endogenous public information). Consequently, for the same fixed parameter values, endogenous public information may overturn the sign of social value of information results with exogenous public information. Furthermore, since $k_1^2 < 1$, reducing the noise in the endogenous public signal changes welfare at a slower rate compared to when public information is exogenous. Corollary 9 shows the effects of disclosing more precise private and public information on total surplus.

Corollary 9. *With both exogenous and endogenous public information, total surplus increases with the precision of the noise in the public and private signals.*

This result has been previously noted by Vives (1990) and Ui and Yoshizawa (2015) with exogenous public information. Therefore, when total surplus is the welfare benchmark, we obtain the same social value of information conclusions with exogenous than with endogenous public information (Figures 3a and 3b).

5.3 Beauty Contest

Morris and Shin (2002) considered a utility function which formalises Keynes' beauty contest metaphor for how financial markets work: Agents are not only concerned about predicting fundamentals but also about outguessing the likely actions of others. An agent's utility function can be expressed as[15]

$$U(q, Q, \sigma_q, \theta) = -(1-r)(q_i - \theta)^2 - r(q_i - Q)^2 + r\sigma_q^2, \tag{18}$$

where[16] $r \in (-1, 1)$. The degree of strategic complementarity is $\alpha = r$. Social welfare is $W(Q, \sigma_q, \theta) = -(1-r)(Q-\theta)^2 + r\sigma_q^2$ with $\alpha^* = 0$. With full information, the equilibrium action is efficient and given by: $k(\theta) = k^*(\theta) = \theta$, which implies that $\phi = 0$.

[15]Morris and Shin's (2002) original utility function was $u_i = -(1-r)(q_i - \theta)^2 - r(L_i - \bar{L})$, where $L_i = \int (q_i - q_j)^2 dj$ and $\bar{L} = \int L_j dj$. It can be shown that this utility function is equivalent to (17).

[16]Morris and Shin (2002) only considered the case $r \in (0, 1)$.

The central bank's research department compiles and discloses public information about the economy's fundamentals which affects an agent's utility function. Suppose that the central bank directly observes the fundamentals with some noise and discloses a public statistic, which is exogenous. In another scenario, the central bank does not directly observe the fundamentals but instead observes the market and makes a forecast about the aggregate action. The public statistic is endogenous[17]. Agents also have access to private information and they set strategies which are contingent on the private and public signals. When public information is endogenous, these strategies could be understood as schedules, such as the ones used in algorithmic trading which are contingent on the central bank's public disclosures of macroeconomic forecasts. Next corollary shows the equilibrium and efficient weights to private information with exogenous and endogenous public information (refer to Figures 2c and 2d).

Corollary 10. *With exogenous public information, agents give an inefficiently low (high) weight to private information when actions are strategic complements (strategic substitutes). With endogenous public information, there exists an $\alpha^0 < 0$ such that $\gamma^m(\alpha^0) = \gamma^*$. Agents underweight private information whenever $\alpha > \alpha^0$ (which occurs if actions are strategic complements ($\alpha > 0$) or if actions are strategic substitutes and $\alpha < 0$ is smaller than the information externality). Agents overweight private information whenever $\alpha^0 > \alpha$ (which occurs when actions are strategic substitutes and $\alpha < 0$ is larger than the information externality).*

Corollary 10 reflects how payoff and information externalities interact. These results are in contrast with the benchmark case of exogenous public information since agents underweight private information for a larger range of α with endogenous than exogenous public information (e.g. Morris and Shin (2002) and Angeletos and Pavan (2007)). In particular, agents may underweight private information when actions are strategic substitutes. Next corollary applies the social value of information results to the beauty contest to both when public information is exogenous ($\nu = 0$) and endogenous ($\nu > 0$).

[17] A similar interpretation has been proposed by Morris and Shin (2005), Baeriswyl (2011) and Bond and Goldstein (2015).

Corollary 11. *Case I: If $\alpha \leq \frac{1}{2}$ then welfare increases with the precision of the noise in the public and private signals.*

Case II: If $\alpha > \frac{1}{2}$ then

i) If $\eta \leq \frac{\tau}{\tau_\epsilon}$ and $\frac{\varphi}{(-\chi)} \leq \frac{\tau}{\tau_\epsilon}$ then welfare increases with the precision of the noise in the public and private signals.

ii) If $\eta > \frac{\tau}{\tau_\epsilon}$ and $\frac{\varphi}{(-\chi)} \leq \frac{\tau}{\tau_\epsilon}$ then welfare increases with the precision of the noise in the private signal while it decreases with the precision of the noise in the public signal.

Corollary 11 confirms that, with endogenous public information, the two regions described by Morris and Shin (2002) are also present. If $\alpha \leq \frac{1}{2}$ then welfare increases with the precision of the noise in both public and private signals, as shown in Figure 3a and 3b. However, if $\alpha > \frac{1}{2}$ and if the ratio of public to private precision is small then welfare increases with the precision of the noise in the private signal but more precision in the noise of the public signal may be detrimental (See Figures 3c and 3d). The payoff parameter configuration of the beauty contest rules out that the *'endogenous precision effect'* dominates the *'partial effect'* and therefore it cannot be that welfare decreases with both the precision of the noise in public and private signals.

Endogenous public information has implications for the transparency debate between Morris and Shin (2002) and Svensson (2006), both of which considered that public information was exogenous. Morris and Shin (2002) highlights the potential detrimental effects of public information, while Svensson (2006) argues that the conditions for which public information is detrimental are empirically implausible since more precise public information is detrimental whenever the precision of the noise in the private signal is *at least* 8 times more precise than the noise in the public signal[18]. Svensson (2006) claims that as central banks and information agencies devote far greater resources to analysing and processing public information than private agents do, it seems empirically implausible that the conditions for which public information are detrimental are observed in reality. With endogenous public information, I find that public information is detrimental whenever the precision of the noise in the private signal is *at least* 3.23 times more precise than the noise in the public signal. Therefore, endogenous public information makes it empirically more plausible that public information is detrimental, thus strengthening the conclusion of Morris and Shin (2002) in the transparency debate.

[18]With exogenous public information, the argument follows because an increase in the precision of the noise in public information is detrimental whenever $(1-\alpha)(2\alpha-1) > \frac{\tau^{exo}}{\tau_\epsilon}$, and $f(\alpha) = (1-\alpha)(2\alpha-1)$ is a parabola which reaches a maximum at $\alpha = \frac{3}{4}$ with corresponding value $f(\frac{3}{4}) = \frac{1}{8}$, and therefore more precise public information is detrimental whenever the precision of the noise in the private signal is *at least* 8 times more precise than the noise in the public signal, or at least $\tau_\epsilon > 8\tau^{exo}$.

Figure 1 shows the results of a calibration exercise which compares the implications for the transparency debate with exogenous and endogenous public information. In the region where more precision in the noise of the public statistic can be detrimental, the graph shows the minimum ratio of private to public information precisions as a function of the degree of strategic complementarity for exogenous and endogenous public information. From the graph we first observe that the minimum ratio of private to public information precisions is lower with endogenous than exogenous public information for all degrees of strategic complementarity. The graph also shows that public information is detrimental whenever the precision of the noise in the private signal is *at least* 3.23 times more precise than the noise in the public signal. Furthermore, we notice that with exogenous public information, the minimum occurs at a larger degree of strategic complementarity with endogenous than exogenous public information (0.82 vs. 0.75, respectively).

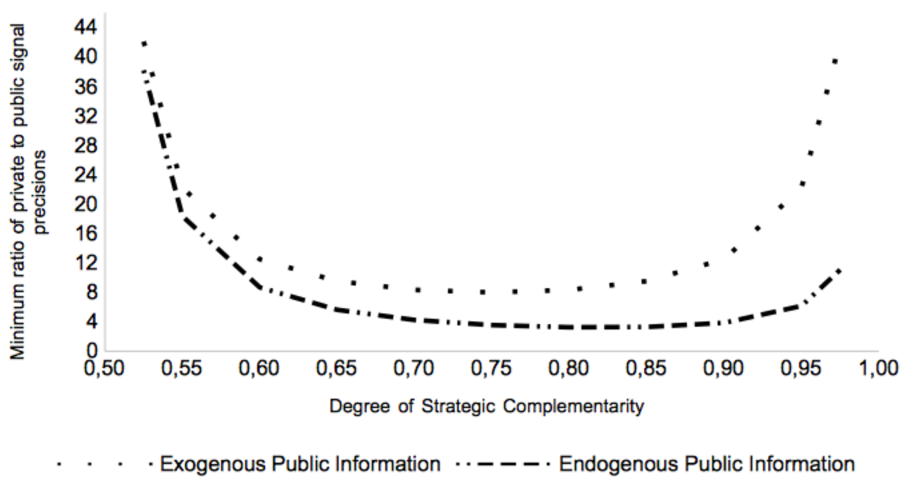

Figure 1: *In the region where more precision in the noise of the public statistic can be detrimental, the graph shows the minimum ratio of private to public information precisions as a function of the degree of strategic complementarity for exogenous and endogenous public information.*

Note: The following fixed parameters have been used for the calibration: $\tau_\theta = 0, \tau_u = \tau_v = \tau_\epsilon = 1$, *and* $k_1 = 1$.

Furthermore, for fixed payoff parameter values and for a fixed precision of the noise in the private signal, welfare decreases for a larger range of precisions of the noise in the public signal, τ_u. While for a constant precision of the noise in private information, increasing the precision of the noise in the endogenous public statistic increases welfare at a faster rate with exogenous than endogenous public information (Corollary 4.i). The implication for public policy is that in order to achieve a certain welfare increase, the precision of the noise in public information needs to be larger with endogenous than exogenous public information.

5.4 Anti-Beauty Contest

The motivation for introducing the anti-beauty contest model is to provide a very stylised game in which actions are strategically independent from an individual perspective, while there is an incentive to coordinate or anti-coordinate at the social level. An agent's utility depends on how close the individual action is to the value of the fundamental and on an aggregate externality which is related to the cross-sectional dispersion of actions of the population. In addition, this game is related to the beauty contest since the relationship between private and social incentives are inverted (and hence the name) and also, if some conditions are specified, this game will turn out to be of the same type of game as competition à la Bertrand with product differentiation but present a more parsimonious structure and therefore easier to analyse. The utility function of the anti-beauty contest game can be expressed as:

$$U(q_i, Q, \sigma_q, \theta) = -(1-r)(q_i - \theta)^2 - r\sigma_q^2. \tag{19}$$

An agent has the objective to take an action close to fundamentals, θ, but there is a negative (positive) payoff externality due to the dispersion in actions of the population when $r > 0$ ($r < 0$), where $-1 < r < 1$. When there is incomplete information, actions are strategically independent since $\alpha = 0$. Welfare under the utilitarian aggregator is given by $W(Q, \sigma_q, \theta) = -(1-r)(Q-\theta)^2 - \sigma_q^2$ and $\alpha^* = r$. The economy is efficient with full information since $k(\theta) = k^*(\theta) = \theta$, which implies that $\phi = 0$.

There is a statistical agency which compiles and disseminates public information. Public information is exogenous whenever the statistical agency uses a fundamental forecasting model in order to form a public statistic using exogenous predictors of fundamentals. Public information is endogenous whenever the statistical agency uses polls and surveys to elaborate the public forecast. In both cases, the aggregation process is noisy. Agents have access to a private signal from their observation of local interactions. Agents' strategies are contingent on both public and private signals. When the public signal is endogenous, agents submit a schedule which is contingent on the public information from the statistical agency. The next corollary shows the relationship between the equilibrium and efficient weights to private information with endogenous and exogenous public information (also refer to Figures 2e and 2f).

Corollary 12. *With exogenous public information, agents place too much (too little) weight on private information if* $\alpha^* > 0$ ($\alpha^* < 0$) *in relation to the efficient level.*

With endogenous public information, there exists an $\alpha^0 > 0$ such that $\gamma^*(\alpha^0) = \gamma^m$. Agents overweight private information whenever $\alpha^* > \alpha^0$ (which occurs when $\alpha^* > 0$ and it is larger than the information externality). Agents underweight private information whenever $\alpha^0 > \alpha^*$ (which occurs if $\alpha^* > 0$ and it is smaller than the information externality or if $\alpha^* < 0$).

Notice that Corollary 12 (anti-beauty contest) presents the inverse result of Corollary 10 (beauty contest). Furthermore, when $\alpha^* = r > 0$, the anti-beauty contest model is structurally equivalent to monopolistic competition à la Bertrand with product differentiation[19] (since $\alpha^* > \alpha \geq 0$ in both models), which in turn presents a similar structure to micro-founded business cycle models where firms set prices. With exogenous public information, models of price setting complementarities find that firms overweight private information in relation to the efficient level (e.g. Hellwig (2005), Adam (2007), Colombo *et al.* (2014)). Corollary 12 implies that this conclusion may be overturned with endogenous public information if the information externality is sufficiently strong. I next discuss the social welfare implications of disclosing more precise public or private information in the anti-beauty contest when public information is exogenous ($\nu = 0$) and endogenous ($\nu > 0$).

Corollary 13. *Case I:If $\alpha^* \leq \frac{1+\upsilon}{2+\upsilon}$ then welfare increases with both public and private information.*
Case II: If $\alpha^* > \frac{1+\upsilon}{2+\upsilon}$, and
i) *If $\frac{\chi}{(-\varphi)} < \frac{\tau_\epsilon}{\tau}$ then welfare increases with both the precision of public information and private information.*
ii) *If $\frac{\chi}{(-\varphi)} > \frac{\tau_\epsilon}{\tau}$ then welfare increases with the precision of public information and decreases with the precision of private information.*

The previous corollary shows that if α^* is less than or equal to $\frac{1+\upsilon}{2+\nu}$, then welfare increases with the precision of both public and private information, as represented by Figures 3a and 3b. However, if α^* is greater than $\frac{1+\upsilon}{2+\nu}$ then welfare always increases with the precision of public information but it may increase (decrease) with the precision of private information if the negative *'partial effect'* is larger (smaller) than the positive *'endogenous public precision effect'*.

Notice that if public information was exogenous the cutoff between Case I and Case II would be equal to $\frac{1}{2}$, while the cutoff with endogenous public information is equal to $\frac{1+\upsilon}{2+\nu}$, which greater than $\frac{1}{2}$. Therefore, with endogenous public information, there is a smaller range of

[19]Competition à la Bertrand with product differentiation may be modelled with a utility function $U(q_i, Q, \sigma_q, \theta) = (\theta - q_i + bQ)q_i - c(c - q_i + bQ)^2$, with $0 < b < 1$. It can be shown that $\alpha^* > \alpha \geq 0$ and $\phi > 0$.

α^* that falls into Case II and a larger range of α^* that falls into Case I than when public information is exogenous.

Figures 3e and 3f illustrate how for the same fixed parameter values, endogenous public information may overturn the sign of social value of information results with exogenous public information due to: (1) the different combination of payoffs that determines the social value of private information ($\frac{(2\alpha^*-1)-\nu(1-\alpha^*)}{(1+\nu(1+\alpha^*))}$ with endogenous public information vs. $2\alpha^*-1$ with exogenous public information), and as a result, a different cutoff value which distinguishes Case I and Case II; (2) the different ratio of public to private information precisions, for $\tau_u = \tau_v$ if $k_1^2\gamma^2 < 1$ then $\frac{\tau^{exo}}{\tau_\epsilon} > \frac{\tau}{\tau_\epsilon}$.

These differences with endogenous and exogenous public information also apply to competition à la Bertrand with product differentiations[20]. Consequently, the results of Corollary 13 are in contrast with the findings of Angeletos and Pavan (2007) and Ui and Yoshizawa (2015) with exogenous public information for competition à la Bertrand. Importantly, I find that for the same fixed parameter values, endogenous public information may overturn the sign of social value of information results with exogenous public information.

6 Concluding Remarks

I have investigated the social value of information with an endogenous public signal in economies which are characterised by payoff externalities and heterogeneous information about fundamentals. In contrast to most of the literature, I have considered that the public signal is a noisy aggregate statistic, and hence is endogenous. This has been motivated by the observation that non-price systems for aggregating information are nowadays ubiquitous, and by the theoretical need to study information externalities which are independent of the market structure. Endogenous public information causes an information externality because an agent in equilibrium does not take into account how his action influences the informativeness of the public signal. As a result, I show that agents place an inefficiently low weight on the private signal over a larger payoff parameter region compared to exogenous public information. In addition, I find that the efficient weight given to private information is larger with endogenous than with exogenous public information.

The way in which payoff and information externalities interact influences the social value of public and private information, which have implications in each application. I have demonstrated that the results of the social value of private information may be overturned when

[20]Using the social value of information categorisation of Ui and Yoshizawa (2015), I find that the anti-beauty contest and the competition à la Bertrand with product differentiation are of the same types of game with the same welfare properties since $\alpha^* > \alpha \geq 0$ and $\phi > 0$.

public information is endogenous in relation to when public information is exogenous. For example this occurs in the anti-beauty contest model. Therefore, the policy implications for determining whether the welfare planner should support policies which make private information about fundamentals more precise depend crucially on the nature of public information (whether exogenous or endogenous).

The type of public information also has an influence on the social value of public information. I have shown than in several applications (e.g. beauty contest, anti-beauty contest or competition à la Cournot with product differentiation) the rate at which welfare changes with respect to the precision of the noise in the public statistic is slower with endogenous than with exogenous public information. The implication for public policy is that in order to achieve a certain welfare increase, the change in the precision of the noise in public information needs to be larger with endogenous than with exogenous public information. Furthermore, I find that it is more likely that public information is detrimental with endogenous than with exogenous public information in the beauty contest, thus having implications for the optimal amount of public information that should be disclosed, and for the social value of public information transparency debate.

This paper could be extended in a number of ways. First, notice that one crucial assumption is that the endogenous public signal is defined as the noisy aggregate action. However, one could also think of public information reflecting a different statistic summarising aggregate data. More importantly, in some contexts the endogenous public signal can be interpreted as the price since it emerges as a result of a market clearing process. Different market microstructures may lead to different definitions of the public signal, which would undoubtedly lead to different results, as in the work of Vives (2013). Therefore, it would be interesting to investigate how different mechanisms of aggregating information affect the use of information and the social value of public and private information. Another issue is how this model relates to other theoretical models which also consider that the information structure is endogenous but for different motives, such as the literature of information acquisition (e.g. Colombo, Femminis and Pavan (2014)) and rational inattention (e.g. Sims (2006) and Veldkamp (2011)). Additionally, one could explore how robust these results are in more general information structures, such as in Myatt and Wallace (2012) or Bergemann and Morris (2013).

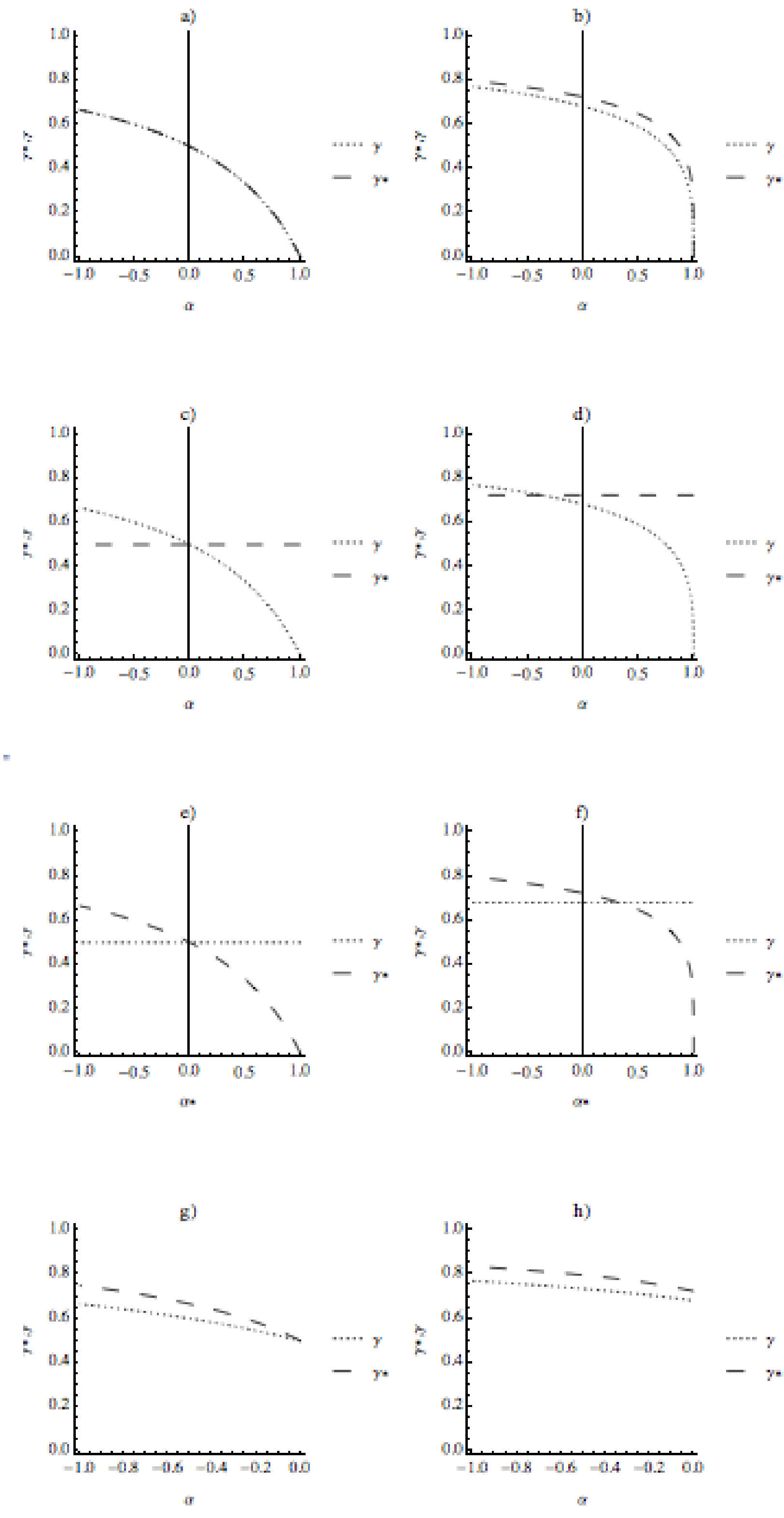

Figure 2: *Equilibrium and efficient weights to private information with exogenous (left) and endogenous (right) public information.* The vertical axis displays the equilibrium, γ, and efficient weight to private information, γ^*, while the horizontal axis corresponds to α, α^*, respectively. Fixed parameters are $\tau_u = \tau_v = \tau_\epsilon = 1$ and $\tau_\theta = 0$. Plots a) and b) correspond firms competing in a homogeneous product market; c) and d) to the beauty contest; e) and f) to firms competing à la Bertrand with product differentiation. ; and g) and h) to firms competing à la Cournot with product differentiation.

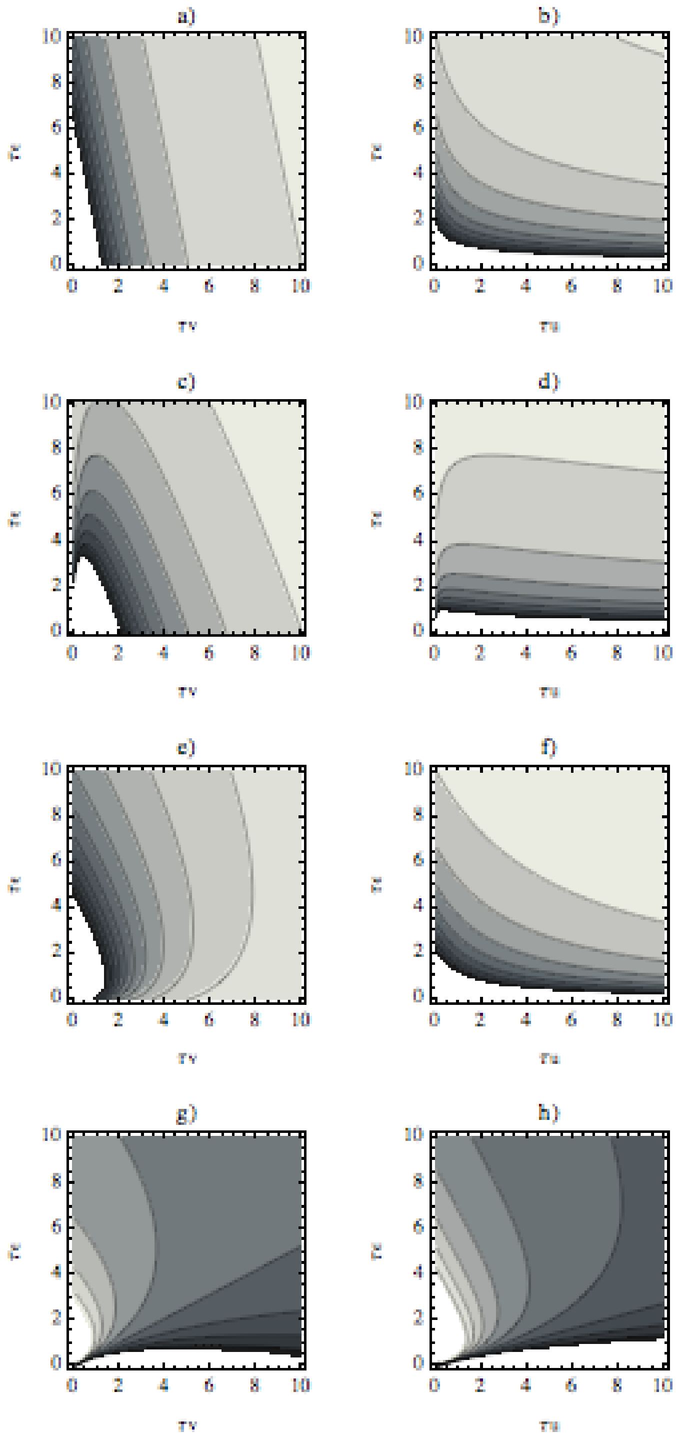

Figure 3: *Social welfare contours with exogenous (left) and endogenous (right) public information.* The horizontal axis displays the precision of the noise in the public statistic (τ_v or τ_u) and the vertical axis displays the precision of the noise in the private signal (τ_ϵ). Welfare is larger (smaller) when the colour is lighter (darker). Fixed parameters are $\tau_u = \tau_v = \tau_\epsilon = 1$ and $\tau_\theta = 0$. Contour plots a) and b) refer to firms competing in a homogeneous product market and to Case I in monopolistic competition à la Cournot and à la Bertrand with product differentiation and in the beauty contest. c) and d) refer to Case II in monopolistic competition à la Cournot and beauty contest. e) and f) refer to Case II in monopolistic competition à la Bertrand. g) and h) refer to Case III in monopolistic competition à la Cournot.

Appendix

Proof of Proposition 1 First, I find the equilibrium strategy with full information, which will satisfy $U_q = 0$. Note that $U_{qq} < 0$ and therefore the second order condition is satisfied. With full information $q_i = Q = k(\theta)$, and since U is quadratic, the full information strategy will be a linear function of fundamentals $k(\theta) = k_0 + k_1\theta$. A Taylor expansion of $U_q(q, q, 0, \theta)$ around $q = 0$ and $\theta = 0$ gives $U_q(k, k, 0, \theta) = U_q(0, 0, 0, 0) + U_{qq}k + U_{qQ}k + U_{q\theta}\theta$. Therefore, $k(\theta) = -\frac{U_q(0,0,0,0)}{U_{qq}+U_{qQ}} - \frac{U_{q\theta}\theta}{U_{qq}+U_{qQ}}$, and notice that $U_{qq} + U_{qQ} \neq 0$ since $\alpha < 1$.

Second, I find the equilibrium strategy with incomplete information and an endogenous public signal. A best response is a strategy q', satisfies $E\left[U_q(q', Q, \sigma_q, \theta) \mid s_i, z\right] = 0$, where a Taylor expansion of U_q around the full information equilibrium is given by: $U_q(q', Q, \sigma_q, \theta) = U_q(k, k, 0, \theta) + U_{qq}(q' - k) + U_{qQ}(Q - k)$, where $U_q(k, k, 0, \theta) = 0$ and $U_{qq} < 0$.

Therefore, using the definition of α, the best response strategy is

$$q'(s_i, z) = E\left[(1 - \alpha)k(\theta) + \alpha Q \mid s_i, z\right]. \tag{A1}$$

An arbitrary strategy is of the form $q(s_i, z) = b + as_i + cE\left[\theta \mid z\right]$, and note that $E\left[\theta|z, s_i\right] = \frac{\tau_\epsilon}{\tau_\epsilon+\tau}s_i + \frac{\tau}{\tau_\epsilon+\tau}E\left[\theta \mid z\right]$. Then, I substitute this expression in the best response strategy, match coefficients to find the fixed point, and I obtain: $b = k_0$, $a = \frac{k_1(1-\alpha)\tau_\epsilon}{(1-\alpha)\tau_\epsilon+a^2\tau_u+\tau_\theta} = k_1\gamma$ and $c = \frac{k_1\tau}{(1-\alpha)\tau_\epsilon+a^2\tau_u+\tau_\theta} = k_1(1 - \gamma)$. Note that the equation for a defines an implicit function of γ, which will be the solution of the cubic equation

$$\pi(\gamma) = \gamma^3 k_1^2\tau_u + \gamma((1 - \alpha)\tau_\epsilon + \tau_\theta) - (1 - \alpha)\tau_\epsilon, \tag{A2}$$

where $\pi(\gamma^m) = 0$. By the Descartes' Rule of signs, I note that there is only one sign change and therefore, there will be only a positive real root. Checking $\pi(-\gamma)$, I notice that there are no sign changes, and therefore, there are no negative real roots. To conclude, this equation will have one positive real root and 2 imaginary real roots. I consider the positive real root. Additionally, I note that $\pi(0) = -(1 - \alpha)\tau_\epsilon < 0$ and $\pi(1) = k_1^2\tau_u + \tau_\theta > 0$, and therefore, the positive real root will be between 0 and 1. Therefore, the weight to private information will be a positive number between 0 and 1. ■

Proof of Corollary 1 Differentiating $\pi(\gamma^m)$ with respect to τ_ϵ, I obtain that

$$\frac{\partial\gamma^m}{\partial\tau_\epsilon} = \frac{(1 - \alpha)(1 - \gamma^m)}{(1 - \alpha)\tau_\epsilon + \tau_\theta + 3(\gamma^m)^2 k_1^2\tau_u} > 0, \tag{A3}$$

since $0 < \gamma^m < 1$. Differentiating $\pi(\gamma^m)$ with respect to τ_θ,

$$\frac{\partial\gamma^m}{\partial\tau_\theta} = \frac{-\gamma^m}{(1 - \alpha)\tau_\epsilon + \tau_\theta + 3(\gamma^m)^2 k_1^2\tau_u} < 0, \tag{A4}$$

since $0 < \gamma^m < 1$. Similarly,

$$\frac{\partial \gamma^m}{\partial \tau_u} = \frac{-(\gamma^m)^3 k_1^2}{(1-\alpha)\tau_\epsilon + \tau_\theta + 3(\gamma^m)^2 k_1^2 \tau_u} < 0. \tag{A5}$$

and differentiating $\pi(\gamma^m)$ with respect to α:

$$\frac{\partial \gamma^m}{\partial \alpha} = \frac{-(1-\gamma^m)\tau_\epsilon}{(1-\alpha)\tau_\epsilon + \tau_\theta + 3(\gamma^m)^2 k_1^2 \tau_u} < 0, \tag{A6}$$

since $0 < \gamma^m < 1$, then $\frac{\partial \gamma^m}{\partial \alpha} < 0$. ■

Proof of Proposition 2 First, I find the efficient strategy with full information, $k^*(\theta)$, which satisfies $W_Q(k^*, 0, \theta) = 0$. The second order condition is satisfied since $W_{QQ} < 0$. A Taylor expansion of W_Q around $k^*(\theta) = 0$ gives $W_Q(k^*, 0, \theta) = W_Q(0, 0, 0) + W_{QQ}k^* + W_{Q\theta}\theta = 0$. By the same argument as in Proposition 1, $k^*(\theta)$ will be a linear function of θ, which can be written as $k^*(\theta) = k_0^* + k_1^*\theta$. Therefore, $k^*(\theta) = -\frac{W_Q(0,0,0)}{W_{QQ}} - \frac{W_{Q\theta}\theta}{W_{QQ}}$.

Second, Angeletos and Pavan (2007) show that the expression for $E[u]$ for a candidate equilibrium strategy is

$$E[u] = E[W(k(\theta), 0, \theta)] + E[W_Q(k(\theta), 0, \theta)(Q - k(\theta))] + \frac{W_{QQ}}{2} E\left[(Q - k(\theta))^2\right] + \frac{W_{\sigma\sigma}}{2} E\left[(Q - q)^2\right]. \tag{A7}$$

Note that $Q - k(\theta) = k_1(1-\gamma)(E[\theta \mid z] - \theta))$ and $q - Q = k_1\gamma\epsilon_i$. The term $E[W_Q(k(\theta), 0, \theta)(Q - k(\theta))]$ is more involved. A Taylor expansion of $W_Q(k(\theta), 0, \theta))$ around $k(\theta) = k^*(\theta)$ is equivalent to $W_Q(k(\theta), 0, \theta)) = W_Q(k^*(\theta), 0, \theta)) + W_{QQ}(k(\theta) - k^*(\theta))$, which is equal to $W_{QQ}(k(\theta) - k^*(\theta))$ since $W_Q(k^*(\theta), 0, \theta)) = 0$. Since $E[(E[\theta \mid z] - \theta), \theta] = \frac{-1}{\tau}$ and $W_{QQ} < 0$, the expectation term is equal to

$$E[W_Q(k(\theta), 0, \theta)(Q - k(\theta))] = \frac{- \mid W_{QQ} \mid k_1^2 \phi(1-\gamma)}{\tau}, \tag{A8}$$

where the full information inefficiency is $\phi = \frac{k_1^* - k_1}{k_1}$. Therefore, ex ante utility at a candidate equilibrium strategy is equivalent to

$$E[u] = E[W(k(\theta), 0, \theta)] - \frac{\mid W_{\sigma\sigma} \mid k_1^2}{2} \left(\frac{\gamma^2}{\tau_\epsilon} + \frac{(1-\alpha^*)(1-\gamma)^2}{\tau} + \frac{2(1-\alpha^*)\phi(1-\gamma)}{\tau} \right). \tag{A9}$$

Notice that maximising ex ante utility is equivalent to minimising welfare losses at a candidate equilibrium strategy, which can be written as

$$WL(\gamma) = \frac{\mid W_{\sigma\sigma} \mid k_1^2}{2} \left(\frac{\gamma^2}{\tau_\epsilon} + \frac{(1-\alpha^*)(1-\gamma)^2}{\tau} + \frac{2(1-\alpha^*)\phi(1-\gamma)}{\tau} \right). \tag{A10}$$

Third, I find the welfare planner's efficient strategy with incomplete information and an endogenous public signal. Note that the welfare planner internalizes all externalities, including payoff externalities with full information, which imply that $k^*(\theta) = k(\theta)$ and hence $\phi = 0$. The first order condition can be computed as $\frac{dWL}{d\gamma} = \left(\frac{\partial WL}{\partial \gamma}\right)_{\tau\, const} + \frac{\partial WL}{\partial \tau}\frac{\partial \tau}{\partial \gamma} = 0$, and hence $\frac{dWL}{d\gamma} = 0$ at $\gamma = \gamma^*$ and $WL(\gamma)$ is a strictly quasiconvex function of γ. The first order condition is

$$\left(\frac{\gamma^*}{\tau_\epsilon} - \frac{(1-\alpha^*)(1-\gamma^*)}{\tau^*} - \frac{(k_1^*)^2\tau_u\gamma^*(1-\alpha^*)(1-\gamma^*)^2}{(\tau^*)^2}\right) = 0. \tag{A11}$$

which defines γ^* implicitly as $\gamma^* = \frac{(1-\alpha^*)\tau_\epsilon}{(1-\alpha^*)\tau_\epsilon + \tau^* - (1-\alpha^*)(1-\gamma^*)^2\frac{\tau_u\tau_\epsilon(k_1^*)^2}{\tau^*}}$, and γ^* is defined as a solution of a quintic equation which can be written as $\psi(\gamma^*) = 0$, where

$$\psi(\gamma) = (\gamma)^5\tau_u^2k_1^4 + 2(\gamma)^3k_1^2\tau_u\tau_\theta + (\gamma)^2k_1^2\tau_u\tau_\epsilon(1-\alpha^*) + \gamma(\tau_\theta^2 + \tau_\epsilon(1-\alpha^*)(\tau_\theta - \tau_uk_1^2)) - \tau_\epsilon\tau_\theta(1-\alpha^*). \tag{A12}$$

Applying Descartes' Rule of signs to find out the number of real roots, I notice that there is only one change in sign, and therefore, this polynomial will have only one real positive root. Notice however that since $\frac{dWL(\gamma=0)}{d\gamma} < 0$ and therefore $\gamma^* > 0$ and negative roots need not be considered. Furthermore, $\psi(0) = -\tau_\epsilon\tau_\theta(1-\alpha^*) < 0$, while $\psi(1) = \tau_u^2k_1^4 + 2k_1^2\tau_u\tau_\theta + \tau_\theta^2 > 0$. Therefore, the unique positive real root will be between 0 and 1. ■

Proof of Corollary 2 Using $\chi(\gamma)$ to derive the derivative of $\frac{\partial\gamma^*}{\partial\tau_\epsilon}$ I note that:

$$\frac{\partial\gamma^*}{\partial\tau_\epsilon} = \frac{\gamma^*(1-\gamma^*)(1-\alpha^*)(\tau_uk_1^2)}{5(\gamma^*)^4\tau_u^2k_1^4 + 6(\gamma^*)^2k_1^2\tau_u\tau_\theta + 2\gamma^*k_1^2\tau_u\tau_\epsilon(1-\alpha^*)} > 0, \tag{A13}$$

since $0 < \gamma^* < 1$. And

$$\frac{\partial\gamma^*}{\partial\tau_\theta} = \frac{-2\gamma^*\tau^* + \tau_\epsilon(1-\alpha^*)(1-\gamma^*)}{5(\gamma^*)^4\tau_u^2k_1^4 + 6(\gamma^*)^2k_1^2\tau_u\tau_\theta + 2\gamma^*k_1^2\tau_u\tau_\epsilon(1-\alpha^*)} < 0, \tag{A14}$$

can be easily shown to be negative by the definition of γ^*. I find that

$$\frac{\partial\gamma^*}{\partial\tau_u} = k_1^2\gamma^*\left(\frac{-2(\gamma^*)^2\tau^* + \tau_\epsilon(1-\alpha^*)(1-\gamma^*)}{5(\gamma^*)^4\tau_u^2k_1^4 + 6(\gamma^*)^2k_1^2\tau_u\tau_\theta + 2\gamma^*k_1^2\tau_u\tau_\epsilon(1-\alpha^*)}\right), \tag{A15}$$

whose sign of this is is ambiguous. Finally,

$$\frac{\partial\gamma}{\partial\alpha^*} = \frac{-(\gamma\tau_\epsilon\tau_uk_1^2 + \tau_\epsilon\tau_\theta)(1-\gamma^*)}{5(\gamma^*)^4\tau_u^2k_1^4 + 6(\gamma^*)^2k_1^2\tau_u\tau_\theta + 2\gamma^*k_1^2\tau_u\tau_\epsilon(1-\alpha^*)} < 0, \tag{A16}$$

since $0 < \gamma^* < 1$. ■

Proof of Proposition 3 Under the conditions defined in the Proposition 2, WL is strictly quasiconvex function of γ, and $\frac{WL(\gamma)}{d\gamma} = 0$ at $\gamma = \gamma^*$. Therefore $sign\left(\frac{dWL}{d\gamma}\mid_{\gamma=\gamma^m}\right) =$

$sign\,(\gamma^m - \gamma^*)$, which implies that

$$sign\,(\gamma^m - \gamma^*) = sign\left(\alpha^* - \alpha - \frac{(1-\alpha^*)(1-\alpha)(k_1^2\tau_u\tau_\epsilon)}{((1-\alpha)\tau_\epsilon+\tau)^2}\right), \tag{A17}$$

where the information externality is $\triangle = \frac{(1-\alpha^*)(1-\alpha)(k_1^2\tau_u\tau_\epsilon)}{((1-\alpha)\tau_\epsilon+\tau)^2} < 0$. For the three cases considered, note that:

If $\alpha = \alpha^*$ then $sign\,(\gamma^m - \gamma^*) = sign\left(-\frac{(1-\alpha^*)(1-\alpha)(k_1^2\tau_u\tau_\epsilon)}{((1-\alpha)\tau_\epsilon+\tau)^2}\right) < 0$.

Now suppose that $\alpha^*(\alpha) = \xi\alpha$, where $\xi \geq 0$ and $\xi \neq 1$. Notice that the equilibrium and efficient weights to private information are both strictly decreasing function of α, since $\gamma(\alpha)$ and $\gamma^*(\alpha^*(\alpha))$, which range from 1 to 0 as $\alpha \to -\infty$ to $\alpha \to 1$ and $\alpha^* \to 1$, respectively. Therefore, using the intermediate value theorem, I can show that there exists a unique intersection between the two curves, α^0, which is equal to $\alpha^0 = \frac{(1-\alpha^*)(1-\alpha)(k_1^2\tau_u\tau_\epsilon)}{(\xi-1)((1-\alpha)\tau_\epsilon+\tau)^2}$ and satisfies $-\infty < \alpha^0 < 1$. Notice that

$$sign(\alpha^0) = sign(\frac{(1-\alpha^*)(1-\alpha)(k_1^2\tau_u\tau_\epsilon)}{(\xi-1)((1-\alpha)\tau_\epsilon+\tau)^2}) = sign(\xi - 1). \tag{A18}$$

Using the result of the first part of the Corollary, it follows immediately that

$$sign\,(\gamma - \gamma^*) = sign\left(\alpha(\xi-1) - (\xi-1)\alpha^0\right) = sign\left((\xi-1)(\alpha-\alpha^0)\right), \tag{A19}$$

and results follow. ■

Proof of Corollary 3 i) This is an immediate consequence of the first part of Proposition 3.

ii) Note that γ^* is the solution of $\frac{dWL}{d\gamma} = \frac{\partial WL}{\partial\gamma}\mid_{\tau=const} + \frac{\partial WL}{\partial\tau}\frac{\partial\tau}{\partial\gamma} = 0$, which implies that $\frac{\partial WL}{\partial\gamma}\mid_{\tau=const} > 0$ since and $\frac{\partial WL}{\partial\tau}\frac{\partial\tau}{\partial\gamma} < 0$ (because $\frac{\partial WL}{\partial\tau} < 0$ and $\frac{\partial\tau}{\partial\gamma} > 0$). Also note that γ^{*exo} is the solution of $\frac{\partial WL}{\partial\gamma}\mid_{\tau=const}$. Therefore, because WL is strictly quasiconvex, then $\gamma^* > \gamma^{*exo}$. ■

Proof of Proposition 4 Comparative statics of ex ante utility with respect to τ_u are given by (12), which are equivalent to the opposite comparative statics of equilibrium welfare loss given by (9) with respect to τ_u. I obtain the following two expressions

$$\left(\frac{\partial WL(\gamma^m)}{\partial\tau}\right)_{\gamma\,cons.} = \frac{\mid W_{\sigma\sigma}\mid (k_1)^2}{2}\left(\frac{(1-\alpha)((2\alpha-\alpha^*-1)-2\phi(1-\alpha^*))\tau_\epsilon - (1-\alpha^*)(1+2\phi)\tau}{(\tau_\epsilon(1-\alpha)+\tau)^3}\right), \tag{A20}$$

and

$$\frac{d\tau}{d\tau_u}=k_1^2(\gamma^m)^2+2k_1^2\tau_u\gamma^m\frac{\partial\gamma^m}{\partial\tau_u}=k_1^2\gamma^2(\frac{(1-\alpha)\tau_\epsilon+\tau_\theta+(\gamma^m)^2k_1^2\tau_u}{(1-\alpha)\tau_\epsilon+\tau_\theta+3(\gamma^m)^2k_1^2\tau_u}). \tag{A21}$$

Notice that $0<\frac{d\tau}{d\tau_u}<k_1^2(\gamma^m)^2$. Therefore, $\frac{d(E[u(\gamma^m)])}{d\tau_u}\geq 0$ if and only if

$$\eta\leq\frac{\tau\varsigma}{\tau_\epsilon}, \tag{A22}$$

where $\eta=\frac{(1-\alpha)(2\alpha-\alpha^*-1-2\phi(1-\alpha^*))}{(1-\alpha^*)}$ *and* $\varsigma=(1+2\phi)$.■

Proof of Proposition 5 Comparative statics of ex ante utility with respect to τ_ϵ are given by (14), which are equivalent to the opposite comparative statics of equilibrium welfare loss given by (9) with respect to τ_ϵ. I obtain the following expressions:

$$\left(\frac{\partial WL(\gamma^m)}{\partial\tau_\epsilon}\right)_{\tau\, cons.}=\Lambda\left(\frac{-(1-\alpha)(1-\alpha+2\phi(1-\alpha^*))\tau_\epsilon+(2\alpha^*-\alpha-1-2\phi(1-\alpha^*))\tau}{(\tau_\epsilon(1-\alpha)+\tau)^3}\right), \tag{A23}$$

where $\Lambda=\frac{|W_{\sigma\sigma}|(k_1)^2(1-\alpha)}{2}$. We also have that

$$\nu=\frac{d\tau}{d\tau_\epsilon}=(2k_1^2\tau_u\gamma^m\frac{\partial\gamma^m}{\partial\tau_\epsilon})=\frac{2k_1^2\tau_u(1-\alpha)(1-\gamma^m)\gamma^m}{(1-\alpha)\tau_\epsilon+\tau_\theta+3(\gamma^m)^2k_1^2\tau_u}>0 \tag{A24}$$

Combining the previous two expressions, I obtain that $\frac{d(E[u(\gamma^m)])}{d\tau_u}\geq 0\iff\varphi\tau_\epsilon+\chi\tau\leq 0$, where

$$\varphi=-(1-\alpha)^2(1-\alpha+2\phi(1-\alpha^*))+\upsilon(1-\alpha)(2\alpha-1-\alpha^*-2\phi(1-\alpha^*)) \tag{A25}$$

$$\chi=(1-\alpha)(2\alpha^*-\alpha-1)-2\phi(1-\alpha)(1-\alpha^*)-\upsilon(1-\alpha^*)(1+2\phi) \tag{A26}$$

■

Proof of Corollary 4 i) From Proposition 4, we note that if $k_1^2\leq 1$ then $\frac{d\tau}{d\tau_u}\leq 1$ which implies that $\frac{d(E[u(\gamma^m)])}{d\tau_u}\leq\left(\frac{\partial(E[u(\gamma^m)])}{\partial\tau_u}\right)_{\gamma\, cons.}$.

ii) Direct application of Proposition 5.

iii) It follows directly from the definition of $\tau=\tau_\theta+k_1^2\gamma^2\tau_u$ and $\tau^{exo}=\tau_\theta+\tau_v$. ■

Proof of Corollaries 5, 7, 10 and 12 Direct application of Proposition 3 to the payoff structure of each application.

Proof of Corollary 6, 8, 10, 11 and 13. Direct application of Propositions 4 and 5 to the payoff structure of each application.

In Corollary 6, in the application of firms competing in a homogeneous product market, we have that $\eta < 0$, $\varsigma = 1$ and $\varphi < 0$, $\chi < 0$.

In Corollary 8, in monopolistic competition à la Cournot with product differentiation with total profits as welfare benchmark, $\eta = \frac{-(1-\alpha)(1+2\alpha)}{(1-2\alpha)}$, $\varsigma = \frac{1}{1-2\alpha} > 0$ and $\varphi = -((1-\alpha)^2(1+\alpha) + \upsilon(1-\alpha)(1+2\alpha))$, $\chi = -(\upsilon + (1-\alpha)^2) < 0$. Case I has $\eta \leq 0$ and $\varphi < 0$; Case II has $\eta > 0$ and $\varphi < 0$, while Case III has $\eta > 0$ and $\varphi > 0$.

In Corollary 9, in monopolistic competition à la Cournot with product differentiation with total surplus as welfare benchmark,$\eta < 0$, $\varsigma > 0$ and $\varphi < 0$, $\chi < 0$.

In Corollary 11, in the beauty contest application, we have that $\eta = (1-\alpha)(2\alpha-1)$, $\varsigma = 1$ and $\varphi = -(1-\alpha)^3 + \upsilon(1-\alpha)(2\alpha-1)$, $\chi = -(1-\alpha^2) - \upsilon < 0$. Case I is characterised by $\eta \leq 0$; $\varphi < 0$, while Case II is characterised by $\eta > 0$ and $\varphi < 0$.

In Corollary 13, in the anti-beauty contest, $\eta = \frac{-(1+r)}{(1-r)} < 0$, $\varsigma = 1$ and $\varphi = -\upsilon(1+r) < 0$, $\chi = (2r-1) - \upsilon(1-r)$. Case I has $\chi < 0$ and Case II has $\chi > 0$.

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